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A dynamic model of quality competition with endogenous prices

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Abstract

We develop a dynamic model of price and quality competition in order to analyse the effects of competition intensity on quality provision and to which extent an unregulated market is able to provide a socially optimal quality level. Using a differential-game approach with price and quality competition on a Hotelling line, we compare the benchmark open-loop solution against the feedback closed-loop solution, which implies strategic dynamic interaction over time. We find that steady-state quality in the closed-loop solution is (i) increasing in the degree of competition between firms, (ii) lower than in the open-loop solution, and (iii) lower than the socially optimal level. In contrast, steady-state quality in the open-loop solution is at the socially optimal level and independent of competition intensity. Thus, our analysis identifies dynamic strategic interactions between competing firms as an independent source of inefficiency in quality provision.

Keywords: Competition; Quality; Differential-games.

JEL: H42; I11; L13.

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1 Introduction

In many industries, quality is a highly important aspect of the goods or services offered, which in turn affects the way firms compete. If consumers make their purchasing decisions partly based on quality, a firm can attract more consumers not only by lowering the price of its product, but also by increasing its quality. However, since a firm's incentive for attracting more demand by providing higher quality is positively related to the price of the product offered, price and quality decisions tend to interact in a way that makes the effect of competition on quality generally ambiguous. It is therefore of theoretical interest to analyse which factors can potentially determine whether competition has a positive or negative impact on quality provision.

Whether competition stimulates or stifles quality provision is also a question of great interest for policy makers, particularly in sectors like health care, long-term care, education and child care, where quality is a key issue. In these industries, prices tend to be regulated in some countries and unregulated in others.[1] There are several issues that are relevant for the question of whether prices should be regulated or not. One important issue is how free pricing will affect quality provision, and whether competition along both dimensions (price and quality) will lead to a socially optimal quality provision or not.

In this paper we revisit the question of how competition affects quality in a dynamic context, where quality provision requires investments and where quality is treated as a stock that can be increased over time only if the investment in quality is higher than its deterioration. This is a highly relevant feature of many dimensions of quality, since increased quality might require investments in new machinery and additional training of the firm's workforce, for example. We take the dynamic aspect of quality provision into account by developing a model of price and quality competition within a Hotelling framework, where two horizontally differentiated firms choose prices and quality investments in each period of an infinitely repeated game.

We use a differential-game approach to derive the equilibrium price and quality provision.[2]

[1] See Brekke et al. (2014) for a more detailed discussion of price and quality competition in these sectors.

[2] Price competition in oligopoly models, taking a differential game approach, is studied in Vives (1985), Qiu (1997), Driskill and McCafferty (1989), Colombo and Labrecciosa (2015) among many others.

Two different solution concepts are considered, corresponding to two different assumptions regarding the information set available to the players. As a benchmark for comparison, we first derive the *open-loop* solution, where players are assumed to know the initial state (i.e., the initial quality stocks of the firms) but do not (or cannot) observe the evolution of states over time. This implies that each player has to decide its optimal dynamic plan at the beginning of the game and then sticks to it forever. We compare this benchmark with a *closed-loop* solution, where each player can observe the dynamic evolution of states and therefore react to changes in the quality stock of the competitor. More specifically, we derive the closed-loop *feedback* solution, where the players' decisions at each point in time depend on the current state (which summarises the entire history of the game). In contrast to the open-loop benchmark, the closed-loop solution is strongly time-consistent and implies dynamic strategic interaction between the players.

Our analysis produces three main results. First, steady-state quality in the closed-loop solution is increasing in the degree of competition, as measured by a reduction in transportation costs along the Hotelling line. This is in contrast to the open-loop solution, where steady-state quality does not depend on the degree of competition, as would be the case in an equivalent static game. Second, we find that steady-state quality is lower in the closed-loop than in the open-loop solution. The reason is that, in the former case, each firm has an incentive to reduce current quality investments in order to dampen future price competition. This incentive is absent in the open-loop solution, where the players do not interact strategically over time. Third and finally, we find that quality provision is socially optimal in the open-loop solution, which implies that the closed-loop solution is characterised by underprovision of quality in steady state.

The second and third of the above-mentioned results have an interesting parallel in the difference between simultaneous-move and sequential-move versions of an equivalent one-shot game. In a standard symmetric one-shot spatial competition model with price and quality competition, equilibrium quality is at the socially optimal level if the firms make quality and price decisions simultaneously, whereas a sequential-move version of the game – where the firms can commit to quality choices before they set prices – yields lower, and therefore sub-optimal, quality provision in equilibrium.[3] The mechanism is similar to the one giving rise to different

[3] Ma and Burgess (1993) derive this result in the context of a Hotelling model, while Economides (1993) derive

steady-state quality levels in the open-loop and closed-loop solutions of the dynamic model analysed in the present paper. Our analysis can therefore be seen as giving additional support to the sequential-move assumption in one-shot games. Even if price and quality choices are made simultaneously in each period of the game, dynamic strategic interaction (as in the closed-loop solution) will create the same type of incentives for underprovision of quality as in a one-shot game with sequential moves.

Our paper also contributes in a wider sense to the theoretical literature on the relationship between competition and quality. Theoretically, a higher degree of competition has two counteracting effects on quality provision: (i) more competition increases the incentives to provide quality for given prices, but (ii) more competition also reduces the price-cost margin, which in turn reduces the incentives for quality provision. Using the transportation cost parameter as an inverse measure of competition intensity, standard spatial competition models produce a well-known 'neutrality' result, where the two aforementioned effects exactly cancel each other out, and competition intensity has no effect on equilibrium quality provision.[4] Brekke et al. (2010) have shown that this neutrality result is broken in the presence of income effects (where price changes affect the marginal utility of consumers), which creates a positive relationship between competition intensity and quality provision. The present analysis identifies another factor which breaks this netrality result, namely dynamic strategic interaction (as in the closed-loop solution).

The relationship between competition and quality is closely related to the question of whether an unregulated market will produce a socially optimal quality provision. Our analysis also contributes towards answering this question. In a seminal paper, Spence (1975) showed that a monopolist will provide a quality level that is higher (lower) than the socially optimal level if the marginal valuation of quality is higher (lower) for the marginal than for the average consumer. In our model the demand system is linear, which implies that the marginal willingness-to-pay for quality is equal for the marginal and average consumer. In spite of this, steady-state quality provision is socially sub-optimal in the closed-loop solution. Thus, we show that dynamic

the equivalent result in the context of a Salop model.

[4] See, e.g., Ma and Burgess (1993) for the case of competition on a Hotelling line, and Gravelle (1999) for the case of competition on a Salop circle.

strategic interaction between competing firms creates an additional inefficiency that leads to underprovision of quality.

Our work also relates to studies which employ a differential-games approach. Piga (1998, 2000) analyses oligopolistic markets in which firms set price and advertising levels. Advertising has some characteristics that are similar to quality, and can be interpreted as a tool to increase the perceived product quality. However, the way advertising is modelled in these two studies is distinctly different from the way quality competition is modelled in the present paper. Importantly, advertising is modelled as a public good that increases market size. In contrast, quality investments have a purely business-stealing effect in our model. In Piga's models, the ranking of desirability of the outcomes depend on the information rule adopted (open-loop vs feedback).[5] Cellini et al. (2008) focus on persuasive advertising and compare the outcomes of price and quantity competition, and reach the conclusion that price competition entails more advertising.

Brekke et al. (2010) provide a model where oligopolistic firms set qualities in the presence of regulated prices, and constant market size. Quality is also modelled as a stock variable and a Hotelling framework is used. They show that quality is lower under the closed-loop solution than under the open-loop solution when the marginal cost of production is increasing. In contrast, the two solution concepts yield identical quality provision when the marginal cost of production is constant. In the current study we also find that quality is lower under the closed-loop solution. Critically, this result is obtained under a constant marginal cost assumption and is due to the endogenous price (which is instead regulated in the previous study).[6]

Siciliani et al. (2013) consider a model with motivated providers and sluggish demand – which are sensible assumptions in markets in which quality competition is important and prices are regulated, like health care or education. In these models, the strategic nature of quality competition depends on the exact assumptions regarding production costs, with different implications concerning the private and social desirability of outcomes under different information

[5] Like in the current study, Piga (1998) applies a Hotelling framework but, differently, market size (and not quality) is the state variable, which evolves over time according the amount of advertising undertaken by the two firms. In contrast, Piga (2000) presents a model with price as the state variable, in line with the assumption that prices are sticky.

[6] An analogous result is obtained by Brekke et al. (2012) when demand is modelled as sluggish and quality can be changed instantaneously under a fixed price regime.

rules. The presence of provider motivation induces quality to be higher under the closed-loop solution when motivation is modelled as providers caring about the total (gross) surplus of the consumers served by the provider.

Finally, investment in R&D which affects the production cost or product characteristics – and has some parallels to investment in product quality – are studied by Hinloopen (2000, 2003) and Cellini and Lambertini (2005, 2009), among others. Intensity in R&D, and the incentive towards cooperative behaviour, depend on the form of market competition (price vs quantity competition) and the information structure, with a variety of possible outcomes. In general, more intense competition arises when the firms' choice variable is price (rather than quantity), leading to higher consumer surplus in steady-state equilibrium (as is well known, even from static games), and with closed- (rather than open-) loop information structures.

The rest of the paper is organised as follows. In the next section we present the main ingredients of the model. In Section 3 we solve the model under the assumption the players use open-loop decision rules. The open-loop equilibrium is then used as a benchmark for comparison with the closed-loop solution – where the players engage in dynamic strategic interaction – which is analysed in Section 4. The welfare properties of the two solutions are analysed and discussed in Section 5, before the paper is closed with some concluding remarks in Section 6.

2 Model

Consider a market with two firms located at either end of the unit line $S = [0,1]$. On this line segment there is a uniform distribution of consumers, with total mass normalised to 1. Assuming unit demand, the utility of a consumer who is located at $x \in S$ and buys from Firm i, located at $z_i \in \{0,1\}$, is given by

$$U(x, z_i) = v + kq_i - \tau |x - z_i| - p_i, \quad (1)$$

where v is a positive parameter, q_i and p_i are the quality and price, respectively, of the good offered by Firm i, k is a parameter measuring the marginal willingness to pay for quality, and τ is the marginal transportation cost.

Since the distance between firms is equal to one, the consumer who is indifferent between firm i and firm j is located at x_i^D, implicitly given by

$$v - \tau x_i^D + kq_i - p_i = v - \tau \left(1 - x_i^D\right) + kq_j - p_j, \tag{2}$$

and explicitly given by

$$x_i^D = \frac{1}{2} + \frac{k\left(q_i - q_j\right)}{2\tau} - \frac{\left(p_i - p_j\right)}{2\tau}, \tag{3}$$

which is also the demand for Firm i, given the assumptions of (i) uniform consumer distribution (with mass 1), (ii) exogenous locations of providers, and (iii) full market coverage.

We assume that product quality changes over time, due to investment by firms and depreciation. Define $I(t)$ as the investment in quality at time t, and $\delta > 0$ as the depreciation rate of the quality stock. Analytically, the law of motion of quality is given by

$$\frac{dq_i(t)}{dt} := \dot{q}_i(t) = I_i(t) - \delta q_i(t). \tag{4}$$

Each firm has a cost function $C\left(\cdot\right)$, which, at each point in time, depends on the quality investment, the quality stock, and output. For analytical tractability, the cost function is parameterised as follows:

$$C\left(x_i^D, I_i, q_i\right) = cx_i^D + \frac{1}{2}\left(\gamma I_i^2 + \beta q_i^2\right), \tag{5}$$

where $c > 0$, $\gamma > 0$ and $\beta > 0$. Thus, we assume constant marginal cost of production, and increasing and strictly convex costs of quality investments I_i. We also assume that each firm's costs are increasing and convex in the quality stock q_i.

Assuming profit-maximising behaviour, the instantaneous objective function of Firm i is given by

$$\pi_i\left(t\right) = \left(p_i(t) - c\right) x_i^D\left(q_i\left(t\right), q_j\left(t\right), p_i\left(t\right), p_j\left(t\right)\right) - \frac{\gamma}{2} I_i(t)^2 - \frac{\beta}{2} q_i(t)^2, \tag{6}$$

and, defining ρ as the (constant, positive) preference discount rate, the objective function of

Firm i over the infinite time horizon is

$$\int_{0}^{+\infty} \pi_i(t) e^{-\rho t} dt. \tag{7}$$

In the following we model the behaviour of firms, and find the corresponding equilibrium, under two alternative assumptions concerning the information set used by firms at each point of time. First, we model the *open-loop* strategy, where each firm sets its optimal plan at the start of the game, and then sticks to it forever. Under this solution concept, the optimal value of the choice variables simply depends on time (and the value of state variables at the beginning of time). The open-loop solution concept requires minimal amount of information; in some instances, it has been criticised for being 'too static' in nature (Dockner et al., 2000, p. 30); however, in several circumstances, players behave in such a way, especially when the world is difficult to observe or the firms' plans are difficult to modify. The open-loop strategy is usually derived from the solution of a dynamic problem using the Hamiltonian technique. The Nash equilibrium – which is given by the intersection of the plans set by each player – is only weakly time consistent. This is not the case under the *feedback closed-loop* strategy, where the choice variables set by players at any instant of time depends on the (current) value of the states. The feedback strategies are sometimes labelled as 'Markovian', since only the current values of the states matter, irrespective of the past history – which is reflected in the current value of the state vector. The optimal strategies are commonly derived from the solution of Bellman's equation, and the Nash equilibrium under the feedback closed-loop strategy is strongly time consistent.

A large body of theoretical and applied analyses compare the strategy and the equilibrium properties under the two solution concepts. Only in some specific circumstances (see, e.g., Mehlman, 1988, Ch. 4; Dockner et al., 2000, Ch. 7) the two solutions coincide. A variety of outcomes can emerge: while it is impossible, in general, to state which solution concept is individually preferable for players, and which is socially preferable, it is possible to state that the feedback closed-loop solution generally entails a larger degree of competition, since players are able to respond in each point in time to the choice of their opponents.

In these models, it is usual to focus on the steady-state allocation, which can be interpreted as the counterpart of the equilibrium outcome of a static game. As shown below, both the open-loop and the feedback close-loop equilibrium in our model leads the system to a steady state. Given our 'standard' assumptions concerning technology and demand, it is not surprising that the (symmetric) steady state we focus on is stable (in the saddle sense) under the open-loop rule, and it is globally stable under the feedback closed-loop rule.

3 Open-loop solution

Firm i's maximisation problem is given by

$$\underset{I_i(t),\, p_i(t)}{\text{Maximise}} \int_0^{+\infty} \pi_i(t)\, e^{-\rho t} dt, \tag{8}$$

$$\begin{aligned} \text{subject to } \dot{q}_i(t) &= I_i(t) - \delta q_i(t), & (9)\\ \dot{q}_j(t) &= I_j(t) - \delta q_j(t), & (10)\\ q_i(0) &= q_{i0} > 0, & (11)\\ q_j(0) &= q_{j0} > 0. & (12) \end{aligned}$$

Let $\mu_i(t)$ and $\mu_j(t)$ be the current value co-state variables associated with the two state equations. The current-value Hamiltonian is as follows, where time (t) is omitted to ease notation:

$$H_i = (p_i - c)\left(\frac{1}{2} + \frac{k(q_i - q_j)}{2\tau} - \frac{p_i - p_j}{2\tau}\right) - \frac{\gamma}{2} I_i^2 - \frac{\beta}{2} q_i{}^2 - F + \mu_i (I_i - \delta q_i) + \mu_j (I_j - \delta q_j)\,. \tag{13}$$

The solution satisfies the following conditions: (a) $\partial H_i / \partial I_i = 0$, (b) $\partial H_i / \partial p_i = 0$, (c) $\dot{\mu}_i = \rho \mu_i - \partial H_i / \partial q_i$, (d) $\dot{q}_i = \partial H_i / \partial \mu_i$, (e) $\dot{\mu}_j = \rho \mu_j - \partial H_i / \partial q_j$. More extensively, we have:

$$\begin{aligned}
\mu_i &= \gamma I_i, & (14)\\
\frac{1}{2}+\frac{k\left(q_i-q_j\right)}{2\tau}-\frac{p_i-p_j}{2\tau} &= \frac{p_i-c}{2\tau}, & (15)\\
\dot{\mu}_i &= \mu_i\left(\delta+\rho\right)+\beta q_i-\frac{\left(p_i-c\right)k}{2\tau}, & (16)\\
\dot{q}_i &= I_i-\delta q_i, & (17)\\
\dot{\mu}_j &= \left(\delta+\rho\right)\mu_j+\frac{\left(p_i-c\right)k}{2\tau}, & (18)
\end{aligned}$$

to be considered along with the transversality condition $\lim_{t\to+\infty} e^{-\rho t}\mu_i(t)q_i(t)=0$.

The second order conditions are satisfied if the Hamiltonian is concave in the control and state variables (Léonard and Van Long, 1992). This is the case since (i) $H_{I_iI_i} = -\gamma$, (ii) $H_{p_ip_i}=-\frac{1}{\tau}$, (iii) $H_{q_iq_i}=-\beta<0$, (iv) $H_{I_iI_i}H_{q_iq_i}>\left(H_{I_iq_i}\right)^2$, where $H_{I_iq_i}=0$, (v) $H_{I_iI_i}H_{p_ip_i}>0$, and (vi) $H_{p_ip_i}H_{q_iq_i}>0$.

In the steady state we have $\dot{q}_i=0$, $q_i=q_j=q^{OL}$ and $p_i=p_j=p^{OL}$, implying

$$\begin{aligned}
\mu^{OL} &= \gamma I^{OL}, & (19)\\
p^{OL} &= c+\tau, & (20)\\
0 &= \mu^{OL}\left(\delta+\rho\right)+\beta q^{OL}-\frac{k\left(p^{OL}-c\right)}{2\tau}, & (21)\\
I^{OL} &= \delta q^{OL}, & (22)
\end{aligned}$$

which gives

$$p^{OL}=c+\tau \tag{23}$$

and

$$q^{OL}=\frac{k}{2\left(\gamma\delta\left(\delta+\rho\right)+\beta\right)}. \tag{24}$$

It is straightforward to check that the steady state is locally stable in the saddle sense.[7]

[7] Indeed, suppose to evaluate the dynamic system around the steady state, under the symmetry assumption. Firstly, note from (15) that $\dot{p}=0$, so that $p(t)=p^s_{OL}$ and the dynamic system can be reduced to a two-variable system, in I and q. In matrix form this can be written as:

The results are intuitive. The steady-state price equates the sum of marginal production and transportation cost, the latter (τ) being a parameter inversely related to the degree of competition. This result is analogous to the Nash equilibrium of an equivalent static model. Steady-state investment and quality are also decreasing in the marginal cost of quality (β) and investment (γ), and decreasing in the time preference discount rate (ρ). Notice also that a higher depreciation rate of quality (δ) is associated with lower steady-state quality, while the effect on investment can be non-monotonic and depends on the exact parameter configuration.

The most interesting characteristic of the open-loop solution, though, is the independence between marginal transportation costs and steady-state quality. Applying the standard interpretation of τ being an inverse measure of competition intensity, we obtain the following result:

Proposition 1 *When the firms use open-loop decision rules, steady-state quality does not depend on the degree of competition in the market.*

All else equal, stronger competition increases the elasticity of (firm-specific) demand with respect to both price and quality, which leads to lower prices but has two counteracting effects on quality provision: a positive direct effect and an indirect negative effect, since a lower price reduces the incentive to increase quality. In standard spatial competition models, in a static setting, these two effects exactly cancel each other, implying that competition intensity does not affect equilibrium quality provision.[8] Proposition 1 above confirms that this 'neutrality' result carries over to a dynamic setting, as long as the firms use open-loop decision rules. This is perhaps not all that surprising, given the somewhat 'static' nature of the open-loop solution, where the optimal investment plan is decided once and for all at the outset of the game.

$$\begin{bmatrix} \dot{I}(t) \\ \dot{q}(t) \end{bmatrix} = \begin{bmatrix} (\delta+\rho) & \frac{\beta}{\gamma} \\ 1 & -\delta \end{bmatrix} \begin{bmatrix} I(t) \\ q(t) \end{bmatrix} + \begin{bmatrix} -\frac{k}{2\gamma} \\ 0 \end{bmatrix}$$

Clearly, the Jacobian matrix of such a dynamic system has a negative determinant, $-\delta(\delta+\rho) - \beta/\gamma$, and a positive trace, ρ; this means that the steady state is a saddle point. The dynamic properties are the same as in Brekke et al. (2010).

[8] See, e.g., Ma and Burgess (1993) for the case of Hotelling competition and Gravelle (1999) for the case of Salop competition.

4 Closed-loop solution

In this section we present the closed-loop solution, where each firm knows not only the initial state of the system, but can also observe (and therefore react to) the quality stock of the competing firm in all subsequent periods. More specifically, we present the closed-loop *feedback* solution, where the players – at each point in time – make decisions by taking into account the current value of states (which summarises the entire past history of the game). While the closed-loop feedback solution is strongly time-consistent, and therefore arguably a more appealing solution concept in a context of dynamic competition, this solution is also considerably more complicated to calculate. In this section we therefore present directly the optimal dynamic decision rules in the closed-loop feedback solution and relegate the derivation of these rules to the Appendix.

If the parameters β and/or τ are sufficiently large relative to k, which we will henceforth assume is the case, there is a unique globally asymptotically stable closed-loop solution. The optimal pricing rule for Firm i in this solution is given by

$$p_i(t) := \Phi_i^{CL}(q_i(t), q_j(t)) = c + \tau + \frac{k(q_i(t) - q_j(t))}{3}. \tag{25}$$

At each point in time, there is a positive relationship between the quality stock and the price charged by each firm. All else equal, higher quality implies higher demand, which makes demand less price elastic and therefore increases the profit-maximising price. Obviously, an increase in the competitor's quality level has the opposite effect. Since the two firms optimal pricing rules are symmetric, it follows that

$$p_i(t) - p_j(t) = \frac{2k(q_i(t) - q_j(t))}{3}. \tag{26}$$

Thus, at each point in time, the firm with higher quality charges a higher price.

The optimal quality investment rule for Firm i in the closed-loop solution is

$$I_i(t) := \phi^{CL}(q_i(t), q_j(t)) = \frac{1}{\gamma}(\alpha_1 + \alpha_3 q_i + \alpha_5 q_j), \tag{27}$$

where

$$\alpha_1 = \frac{k\gamma}{3\left(\gamma\left(\delta+\rho\right)-\alpha_3\right)} > 0, \tag{28}$$

$$\alpha_3 = s\gamma - \sqrt{\frac{\gamma}{54}\left(4\sqrt{\left(y-2g\right)y}+\left(5y-2g\right)\right)} < 0, \tag{29}$$

and

$$\alpha_5 = -\frac{1}{2}\sqrt{\frac{4\gamma}{27}\left(y-g-\sqrt{y\left(y-2g\right)}\right)} < 0, \tag{30}$$

and where $y := 6\left(s^2\gamma+\beta\right)$, $s := \delta + \frac{1}{2}\rho$ and $g := \frac{k^2}{\tau}$. The negative sign of α_3 is assumed to ensure global asymptotic stability. For the solution to be real, we must also assume that $y \geq 2g$. Finally, given that $\alpha_3 < 0$, the slightly stricter condition $y \geq \frac{8}{3}g$ is sufficient to ensure that the solution is unique. In qualitative terms, the conditions $\alpha_3 < 0$ and $y \geq \frac{8}{3}g$ are both satisfied if β and/or τ are sufficiently large relative to k.

The key property of the quality investment rule given by (27), is the negative sign of α_5, which implies that quality investments are *intertemporal strategic substitutes*[9]; the higher the quality stock of a given firm, the lower the optimal investment level of the competing firm. The intuition for this property is related to the interaction between price and quality investment choices. All else equal, an increase in the quality stock of Firm j leads to reduced demand for Firm i, and this firm will therefore optimally reduce its price, as shown by (25). However, this price reduction implies a lower price-cost margin for Firm i, which in turn implies a reduction in the marginal profit gain of attracting more demand by increasing quality. Firm i will therefore respond by reducing its quality investments.[10]

4.1 Steady state

In steady state, where $q_i = q_j$, equilibrium *prices* in the closed-loop solution are given by

$$p^{CL} = c + \tau \tag{31}$$

[9] As defined by Jun and Vives (2004), intertemporal strategic substitutability implies that the control of each player responds negatively to the state of the other player.

[10] In a static model of price and qualiy competition, Brekke et al. (2015) show that the strategic substitutability of quality choices holds for more general demand functions, and also holds for the case of variable (output-dependent) quality costs, as long as the effect of higher quality on marginal production costs is not too strong.

and are therefore equal to the steady-state prices in the open-loop solution (and to the equilibrium prices in an equivalent static game).

Steady-state *quality* in the closed-loop solution is implicitly given by the steady-state condition $I_i = \delta q_i$, and explicitly given by

$$q^{CL} = \frac{k\gamma}{3\left(\gamma\left(\delta+\rho\right)-\alpha_3\right)\left(\gamma\delta-\left(\alpha_3+\alpha_5\right)\right)}. \tag{32}$$

In addition to $\alpha_3 < 0$, global asymptotic stability also requires $\alpha_3 + \alpha_5 < 0$ and $\alpha_3 - \alpha_5 < 0$. Notice that, since $\alpha_5 < 0$, the condition $\alpha_3 < 0$ ensures that

$$\alpha_3 + \alpha_5 = s\gamma - \sqrt{\frac{\gamma y}{6}} < 0. \tag{33}$$

How does steady-state quality under feedback rules depend on the degree of competition (inversely measured by τ)? Since $\alpha_3 + \alpha_5$ does not depend on τ, it is relatively straightforward to see that

$$\frac{\partial q^{CL}}{\partial \tau} = \underset{+}{\frac{\partial q^{CL}}{\partial \alpha_3}} \underset{+}{\frac{\partial \alpha_3}{\partial g}} \underset{-}{\frac{\partial g}{\partial \tau}} < 0. \tag{34}$$

Thus:

Proposition 2 *When firms adopt feedback (closed-loop) decision rules, steady-state quality is increasing in the degree of competition.*

As previously explained, lower transportation costs have two counteracting effects on the firms' incentives to invest in quality. For given prices, lower transportation costs make demand more quality elastic, which increases the profit-gain of quality investments. On the other hand, lower transportation costs also make demand more price elastic, leading to lower prices, which in turn dampens incentives for quality investments. In contrast to the open-loop case, where these two effects exactly cancel each other in steady state, the first (direct) effect dominates the second (indirect) effect under dynamic competition with feedback rules, yielding a positive relationship between competition intensity and quality provision in steady state.

4.2 Comparison of closed loop and open loop

We have already seen that steady-state prices are equal under both solution concepts. However, steady-state quality provisions differ between the two solution concepts. A comparison (proof in Appendix) yields the following result:

Proposition 3 *Steady-state quality is lower in the closed-loop solution than in the open-loop solution.*

This result is perhaps somewhat surprising. Although higher competition intensity leads to higher steady-state quality levels in the closed-loop solution, as shown in Proposition 2, quality provision is nevertheless always lower in the arguably more 'competitive' strategic environment – when the players use (closed-loop) feedback rules – than in the open-loop setting.

The intuition behind this result is related to how current quality investments affect future price competition. Suppose that, at time t, Firm i has a higher quality level than Firm j (i.e., $q_i(t) > q_j(t)$). The optimal pricing rule, given by (25), then dictates that Firm j should 'compensate' for the lower quality stock by setting a lower price than Firm i. In other words, higher quality investments by one firm today will trigger stronger price competition from the other firm in the future, which – all else equal – dampens the incentives for quality investments. Thus, when the firms use feedback decision rules and can, at each point in time, adjust their investment and price decisions according to the evolution of states, each firm has a strategic incentive to reduce its quality investments in order to dampen future price competition from the rival firm.[11] This is in contrast to the open-loop solution, where there is no strategic interaction over time, and where the above-mentioned strategic effect is not present. This explains why steady-state quality is lower in the closed-loop solution than in the open-loop solution.

The result in Proposition 3 and the intuition behind it has a striking parallel in the difference between simultaneous and sequential decisions in a one-shot version of the game. As shown by Ma and Burgess (1993), equilibrium quality is lower when quality and price decisions are made

[11] Colombo and Labrecciosa (2015) present a differential game of oligopoly, in which a similar mechanism is at work. They consider the case in which firms have to use a renewable productive asset, and show that the decision on current price taken by a player affects the future incentive of opponents to move their own price: this dynamic interdependence can lead the Bertrand competition to be less efficient than Cournot competition.

sequentially rather than simultaneously, and the reason is precisely the strategic incentive to lower quality in order to dampen price competition when quality decisions are made before prices are set.[12] This suggests that, in the case at hand, simultaneous-move and sequential-move games in a static setting provide results which are reasonable parallels of the open-loop and the closed-loop solutions, respectively, in a dynamic setting.

5 Welfare

We define social welfare as the discounted present value of the sum of aggregate consumer surplus and profits accruing over the infinite time horizon. Since total demand is fixed, this is equivalent to aggregate gross consumer utility minus the total costs of production, transportation and quality provision.[13] We derive the first-best optimal solution by letting the social planner choose the quality investment and market share for each firm, in order to maximise social welfare. Formally, this problem is given by

$$\underset{I_i(t),\, I_j(t),\, x_i^D(t)}{\text{Maximise}} \quad W = \int_0^{+\infty} \left[\begin{array}{c} \int_0^{x_i^D(t)} (v - \tau x + kq_i(t))\, dx + \int_{x_i^D(t)}^{1} (v - \tau (1-x) + kq_j(t))\, dx \\ -c - \frac{\gamma}{2} I_i(t)^2 - \frac{\beta}{2} q_i(t)^2 - \frac{\gamma}{2} I_j(t)^2 - \frac{\beta}{2} q_j(t)^2 \end{array} \right] e^{-\rho t} dt, \tag{35}$$

$$\begin{aligned} \text{subject to} \quad \dot{q}_i(t) &= I_i(t) - \delta q_i(t), && (36) \\ \dot{q}_j(t) &= I_j(t) - \delta q_j(t), && (37) \\ q_i(0) &= q_{i0} > 0, && (38) \\ q_j(0) &= q_{j0} > 0, && (39) \end{aligned}$$

[12] Notice that the strategy of reducing quality in order to dampen price competition does not 'succeed' in equilibrium, in the sense that steady-state prices in the closed-loop solution are identical to the ones on the open-loop solution. This is also true for the equivalent simultaneous-move and sequential-move versions of the one-shot game. The reason is of course the symmetric nature of the game, where the effects of unilateral quality reductions on prices are cancelled out in equilibrium, since both firms face exactly the same incentives.

[13] Notice that social welfare does not depend directly on prices, which are here just instruments of surplus distribution between firms and consumers, with no efficiency losses involved.

which simplifies to

$$\underset{I_i(t),\, I_j(t),\, x_i^D(t)}{\text{Maximise}} \quad W = \int_0^{+\infty} \left[\begin{array}{c} v - c - \frac{\tau}{2} + kq_i(t)x_i^D(t) + kq_j(t)\left(1 - x_i^D(t)\right) \\ -\frac{\gamma}{2}I_i(t)^2 - \frac{\beta}{2}q_i(t)^2 - \frac{\gamma}{2}I_j(t)^2 - \frac{\beta}{2}q_j(t)^2 \end{array} \right] e^{-\rho t}dt, \tag{40}$$

subject to (36)-(39).

Let $\mu_i(t)$ and $\mu_j(t)$ be the current value co-state variables associated with the two state equations. The current-value Hamiltonian is:

$$H = v - c - \frac{\tau}{2} + kq_i x_i^D + kq_j\left(1 - x_i^D\right) - \frac{\gamma}{2}\left(I_i^2 + I_j{}^2\right) - \frac{\beta}{2}\left(q_i{}^2 + q_j{}^2\right) + \mu_i\left(I_i - \delta q_i\right) + \mu_j\left(I_j - \delta q_j\right). \tag{41}$$

The solution is given by (a) $\partial H/\partial I_i = 0$, (b) $\partial H/\partial I_j = 0$, (c) $\partial H/\partial x_i^D = 0$, (d) $\dot{\mu}_i = \rho\mu_i - \partial H/\partial q_i$, (e) $\dot{\mu}_j = \rho\mu_j - \partial H/\partial q_j$, (f) $\dot{q}_i = \partial H_i/\partial \mu_i$, (h) $\dot{q}_j = \partial H/\partial \mu_j$, or more extensively,

$$\begin{aligned}
\mu_i &= \gamma I_i, && (42)\\
\mu_j &= \gamma I_j, && (43)\\
k\left(q_i - q_j\right) &= 0, && (44)\\
\dot{\mu}_i &= \left(\rho + \delta\right)\mu_i + \beta q_i - kx_i^D, && (45)\\
\dot{\mu}_j &= \left(\rho + \delta\right)\mu_j + \beta q_j - k(1 - x_i^D), && (46)\\
\dot{q}_i &= I_i - \delta q_i, && (47)\\
\dot{q}_j &= I_j - \delta q_j. && (48)
\end{aligned}$$

In the symmetric steady state we have: $\mu^* = \gamma I^*$, $\left(\rho + \delta\right)\mu^* + \beta q^* - \frac{k}{2} = 0$ and $q^* = \frac{I^*}{\delta}$, which gives

$$q^* = \frac{k}{2\left(\delta\gamma\left(\rho + \delta\right) + \beta\right)} = q^{OL}. \tag{49}$$

Therefore, steady-state quality under open-loop decision rules coincides with the first-best steady-state quality level. Considering the result in Proposition 3, the following result follows immediately:

Proposition 4 *Compared with the first-best optimal level, quality is optimally provided in the open-loop solution and is underprovided in the closed-loop solution.*

The welfare-optimal quality provision in the open-loop solution is partly explained by the linearity of the demand system, which implies that consumers' marginal and average valuations of quality are identical. As demonstrated by Spence (1975) in a monopoly setting, whether quality is over- or under-provided depends on the difference between marginal and average willingness-to-pay for quality. However, dynamic strategic interaction (with feedback decision rules) creates an inefficiency that leads to underprovision of quality in the closed-loop solution.[14]

The welfare properties of the open-loop and closed-loop solutions mimic the welfare properties of the Hotelling model with price and quality competition in a one-shot game, where quality is optimally provided with simultaneous decision making, whereas sequential quality and price decisions imply an underprovision of quality in equilibrium (as shown by Ma and Burgess, 1993). This should come as no great surprise, since we have already established the equivalence between open-loop and closed-loop in a dynamic setting and, respectively, the simultaneous-move and sequential-move versions of the one-shot game. In fact, the properties of the open-loop steady-state solution and the Nash equilibrium of the simultaneous-move static game are virtually identical, including the welfare properties, as indicated by the fact that equilibrium (or steady-state) quality is independent of transportation costs in both cases.

However, a relevant difference occurs between the steady-state quality in the closed-loop solution and the equilibrium quality in the sequential-move one-shot game. Indeed, the quality in the closed-loop dynamic game depends on transportation cost, while equilibrium quality in the sequential-move game does not. Put differently, the 'neutrality' result obtained by static games, according to which lower transportation cost (and hence fiercer competition) has no effect on quality, no longer holds if we consider dynamic competition.

[14] The fact that time, and more specifically competition over time, can be a source of inefficient equilibria is well known in different contexts; for instance, Cellini and Lambertini (1998) show that accumulation of capital over time could be a source of inefficient market allocation in a differential game framework. Again, Araujo and Guimaraes (2015) show that time can be a source of inefficiency in an oligopoly market for the presence of delay options.

6 Conclusions

In this paper we have studied the behaviour of oligopolistic firms, when they can choose the price and the quality of their products. Quality competition in oligopolistic markets is the object of a large body of theoretical and applied literature. The novelty of the present analysis rests on two facts: firstly, we have made the price endogenous, while most available models generally consider prices as regulated when quality is the choice variable (see, e.g., Siciliani et al., 2013). Second, we have taken a differential-game approach, which allows us to highlight how price and quality choices interact when firms make their decisions in a dynamic framework. More specifically, we have presented a differential-game model, under different assumptions concerning the information set used by firms over time, namely, the open-loop rule and the feedback closed-loop rule. The properties of the equilibria generated under these two assumptions are studied and compared with the conclusions provided by static models of price and quality competition (e.g., Ma and Burgess, 1993).

Our model highlights the effect of current quality on rivals' future price decisions, which is shown to play a crucial role in firms' decision making. In particular, steady-state quality emerges to be socially sub-optimal if the closed-loop information rule is used by the competing firms. This is due to the interaction between price and quality in a dynamic setting. Thus, we show that dynamic strategic interaction between competing firms creates an additional inefficiency that leads to underprovision of quality. We have also shown that the steady-state quality in the closed-loop solution is increasing in the degree of competition, as measured by a reduction in transportation costs. This is in contrast to the outcome from an equivalent static game and from the open-loop solution, where the equilibrium quality does not depend on the degree of competition.

In sum, the dynamic approach allows us to uncover relevant and non-trivial effects, both from a positive and a normative point of view. Since quality is generally non-verifiable and thus hard to regulate, and since the under-provision result is caused by dynamic interaction between price and quality choices, our analysis suggests a potential role for price regulation as an instrument that can be used to avoid an inefficient outcome with respect to quality provision.

Our analysis also shows that such a policy intervention might be unnecessary if firms instead are committed to long-term plans regarding quality investments (in which case the relevant solution concept is open-loop). The consideration of time, hence, simply represents an additional source of evaluation in the never-ending debate about the necessity and desirability of public intervention in market economies.

References

[1] Araujo, L., B. Guimaraes, 2015. Intertemporal coordination with delay options. *Journal of Economic Theory*, **157**, 793-810.

[2] Brekke, K., R. Cellini, L. Siciliani, O.R. Straume, 2010. Competition and quality in health care markets: a differential game approach. *Journal of Health Economics*, **29**, 508-523.

[3] Brekke, K., R. Cellini, L. Siciliani, O.R. Straume, 2012. Competition in regulated markets with sluggish beliefs about quality. *Journal of Economics and Management Strategy*, **21**, 131-78.

[4] Brekke, K.R., L. Siciliani, O.R. Straume, 2010. Price and quality in spatial competition. *Regional Science and Urban Economics*, **40**, 471–480.

[5] Brekke, K.R., Siciliani, L., O.R. Straume, 2014. Can competition reduce quality? CEPR Discussion Paper No. 9810

[6] Brekke, K.R., Siciliani, L., O.R. Straume, 2015. Horizontal Mergers and Product Quality. CESIfo Working Paper No. 5406.

[7] Cellini R., L. Lambertini, 1998. A dynamic model of differentiated oligopoly with capital accumulation. *Journal of Economic Theory*, **83**,145-55.

[8] Cellini, R., L. Lambertini, 2005. R&D incentives and market structure: a dynamic analysis. *Journal of Optimization Theory and Applications*, **126**, 85-96.

[9] Cellini, R., L. Lambertini, 2009. Dynamic R&D with spillover: Competition vs. cooperation. *Journal of Economic Dynamics and Control*, **33**, 568-82.

[10] Cellini, R., L. Lambertini, A. Mantovani, 2008. Persuasive advertising under Bertrand competition: a differential game. *Operations Research letters*, **36**, 381-84.

[11] Colombo, L., P. Labrecciosa, 2015. On the Markovian efficiency of Bretrand and Cournot equilibira, *Journal of Economic Theory*, **155**, 332-58.

[12] Dockner, E.J, S. Jørgensen, N. Van Long, G. Sorger, 2000. *Differential Games in Economics and Management Science*, Cambridge, Cambridge University Press.

[13] Driskill R.A., S. McCafferty, 1989. Dynamic Duopoly with adjustment costs: a differential game approach. *Journal of Economic Theory*, **49,** 324-38.

[14] Economides, N., 1993. Quality variations in the circular model of variety-differentiated products. *Regional Science and Urban Economics*, **23**, 235–257.

[15] Gravelle, H., 1999. Capitation contracts: access and quality. *Journal of Health Economics*, **18**, 315–340.

[16] Hinloopen, J., 2000. Strategic R&D Co-operatives. *Research in Economics*, **54**, 153-85.

[17] Hinloopen, J., 2003. R&D efficiency gains due to cooperation. *Journal of Economics*, **80**, 107-25.

[18] Jun, B., X. Vives, 2004. Strategic incentives in dynamic oligopoly. *Journal of Economic Theory*, **116**, 249-81.

[19] Ma, C.A., J.F. Burgess, 1993. Quality competition, welfare and regulation. *Journal of Economics*, **58**, 153-73.

[20] Mehlmann, A., 1988. *Applied differential games*, New York, Plenum Press.

[21] Piga, C.A.G., 1998. A dynamic model of advertising and product differentiation. *Review of Industrial Organization*, **13**, 509-22.

[22] Piga, C.A.G., 2000. Competition in a duopoly with sticky price and advertising. *International Journal of Industrial Organization*, **18**, 595-614.

[23] Qiu, L., 1997. On the dynamic efficinecy of Bertrand and Cournot equilibria. *Journal of Economic Theory*, **75,** 213-29.

[24] Siciliani, L., O.R. Straume, R. Cellini, 2013. Quality competition with motivated providers and sluggish demand. *Journal of Economic Dynamics and Control*, **37**, 2041-71.

[25] Vives, X., 1985. Efficiency of Bertrand and ournot equilibria with product differentiation. *Journal of Economic Theory*, **36**, 166-75.

Appendix

Derivation of the closed-loop solution

The firm's instantaneous objective function is

$$(p_i - c)\left(\frac{1}{2} + \frac{k(q_i - q_j)}{2\tau} - \frac{p_i - p_j}{2\tau}\right) - \frac{\gamma}{2} I_i^2 - \frac{\beta}{2} {q_i}^2 - F \tag{A1}$$

which – faced with the linear dynamic constraint – gives rise to a linear-quadratic problem. Hence, we define the value function as

$$V^i(q_i, q_j) = \alpha_0 + \alpha_1 q_i + \alpha_2 q_j + (\alpha_3/2) q_i^2 + (\alpha_4/2) q_j^2 + \alpha_5 q_i q_j. \tag{A2}$$

Define $I_i = \phi_i(q_i, q_j)$ and $I_j = \phi_j(q_i, q_j)$. The value function has to satisfy the Hamilton-Jacobi-Bellman (HJB) equation:

$$\rho V^i(q_i, q_j) = \max \left\{ \begin{array}{c} (p_i - c)\left(\frac{1}{2} + \frac{k(q_i - q_j)}{2\tau} - \frac{p_i - p_j}{2\tau}\right) - \frac{\gamma}{2} I_i^2 - \frac{\beta}{2} {q_i}^2 - F \\ + V^i_{q_i}(q_i, q_j)(I_i - \delta q_i) + V^i_{q_j}(q_i, q_j)(I_j - \delta q_j) \end{array} \right\}. \tag{A3}$$

Maximisation of the right-hand-side with respect to I_i yields $V^i_{q_i} = \gamma I_i$, which after substitution gives

$$I_i = \phi_i(q_i, q_j) = \frac{\alpha_1 + \alpha_3 q_i + \alpha_5 q_j}{\gamma}. \tag{A4}$$

Similarly, we obtain

$$I_j = \phi_j(q_i, q_j) = \frac{\alpha_1 + \alpha_3 q_j + \alpha_5 q_i}{\gamma}. \tag{A5}$$

Maximisation of the right-hand-side with respect to p_i and p_j yields

$$\begin{aligned} \frac{1}{2} + \frac{k(q_i - q_j)}{2\tau} - \frac{p_i - p_j}{2\tau} - (p_i - c)\frac{1}{2\tau} &= 0, \\ \frac{1}{2} + \frac{k(q_j - q_i)}{2\tau} - \frac{p_j - p_i}{2\tau} - (p_j - c)\frac{1}{2\tau} &= 0, \end{aligned} \tag{A6}$$

from which we obtain the simple expression:

$$\begin{aligned} p_i &= \Phi_i(q_i, q_j) = c + \tau + \frac{k\left(q_i - q_j\right)}{3} \\ p_j &= \Phi_j(q_i, q_j) = c + \tau - \frac{k\left(q_i - q_j\right)}{3} \\ p_i - p_j &= \frac{2k\left(q_i - q_j\right)}{3} \end{aligned} \tag{A7}$$

Substituting $I_i = \phi_i(q_i, q_j)$, $I_j = \phi_j(q_i, q_j)$, $V^i_{q_i}(q_i, q_j) = \alpha_1 + \alpha_3 q_i + \alpha_5 q_j$, $V^i_{q_j} = \alpha_2 + \alpha_4 q_j + \alpha_5 q_i$ into the HJB equation, we obtain

$$\rho V^i(q_i, q_j) = \left\{ \begin{array}{c} \left(\tau + \frac{k(q_i - q_j)}{3}\right)\left(\frac{1}{2} + \frac{k(q_i - q_j)}{6\tau}\right) \\ -\frac{1}{2\gamma}\left(\alpha_1 + \alpha_3 q_i + \alpha_5 q_j\right)^2 - \frac{\beta}{2} {q_i}^2 - F \\ + \left(\alpha_1 + \alpha_3 q_i + \alpha_5 q_j\right)\left(\frac{\alpha_1 + \alpha_3 q_i + \alpha_5 q_j}{\gamma} - \delta q_i\right) \\ + \left(\alpha_2 + \alpha_4 q_j + \alpha_5 q_i\right)\left(\frac{\alpha_1 + \alpha_3 q_j + \alpha_5 q_i}{\gamma} - \delta q_j\right) \end{array} \right\}, \tag{A8}$$

and, after substitution of V^i, we obtain

$$\begin{aligned} &\left(\rho\alpha_0 - \frac{1}{2}\tau - \frac{1}{2\gamma}\alpha_1^2 - \frac{1}{\gamma}\alpha_1\alpha_2\right) \\ &+q_i\left(\alpha_1\left(\delta + \rho\right) - \frac{1}{3}k - \frac{1}{\gamma}\alpha_1\alpha_3 - \frac{1}{\gamma}\alpha_2\alpha_5 - \frac{1}{\gamma}\alpha_1\alpha_5\right) \\ &+q_j\left(\alpha_2\left(\delta + \rho\right) + \frac{1}{3}k - \frac{1}{\gamma}\alpha_2\alpha_3 - \frac{1}{\gamma}\alpha_1\alpha_4 - \frac{1}{\gamma}\alpha_1\alpha_5\right) \\ &+q_i^2\left(\alpha_3\left(\delta + \frac{1}{2}\rho\right) - \frac{1}{2\gamma}\alpha_3^2 - \frac{1}{\gamma}\alpha_5^2 + \frac{1}{2}\beta - \frac{1}{18}\frac{k^2}{\tau}\right) \\ &+q_j^2\left(\alpha_4\left(\delta + \frac{1}{2}\rho\right) - \frac{1}{\gamma}\alpha_3\alpha_4 - \frac{1}{2\gamma}\alpha_5^2 - \frac{1}{18}\frac{k^2}{\tau}\right) \\ &+q_iq_j\left(\left(2\delta + \rho\right)\alpha_5 + \frac{1}{9}\frac{k^2}{\tau} - \frac{2}{\gamma}\alpha_3\alpha_5 - \frac{1}{\gamma}\alpha_4\alpha_5\right) \end{aligned} \tag{A9}$$

For the equality to hold, the terms in brackets in the above equation have to be equal to zero. Notice that the last three terms do not depend on α_0, α_1 and α_2, but only on α_3, α_4 and α_5. We therefore focus on the following system of three equations in three unknowns (α_3, α_4 and

α_5):

$$\begin{aligned} \alpha_3\left(\delta+\frac{1}{2}\rho\right)-\frac{1}{2\gamma}\alpha_3^2-\frac{1}{\gamma}\alpha_5^2+\frac{1}{2}\beta-\frac{1}{18}\frac{k^2}{\tau} &= 0, \\ \alpha_4\left(\delta+\frac{1}{2}\rho\right)-\frac{1}{\gamma}\alpha_3\alpha_4-\frac{1}{2\gamma}\alpha_5^2-\frac{1}{18}\frac{k^2}{\tau} &= 0, \\ \alpha_5\left(2\delta+\rho\right)+\frac{1}{9}\frac{k^2}{\tau}-\frac{2}{\gamma}\alpha_3\alpha_5-\frac{1}{\gamma}\alpha_4\alpha_5 &= 0. \end{aligned} \tag{A10}$$

Define $g := \frac{k^2}{\tau}$ and $s := \left(\delta+\frac{1}{2}\rho\right)$. We can re-write the system more succinctly as:

$$\begin{aligned} s\alpha_3-\frac{1}{2\gamma}\alpha_3^2-\frac{1}{\gamma}\alpha_5^2+\frac{1}{2}\beta-\frac{1}{18}g &= 0, \\ s\alpha_4-\frac{1}{\gamma}\alpha_3\alpha_4-\frac{1}{2\gamma}\alpha_5^2-\frac{1}{18}g &= 0, \\ 2s\alpha_5+\frac{1}{9}g-\frac{2}{\gamma}\alpha_3\alpha_5-\frac{1}{\gamma}\alpha_4\alpha_5 &= 0. \end{aligned} \tag{A11}$$

Define

$$\begin{aligned} A &:= \sqrt{\gamma\left(\frac{3}{2}y-g\right)}>0, \\ B &:= \frac{1}{6}\left(\frac{y}{2}-g\right)y>0, \\ C &:= \frac{4\gamma}{27}\left(y-g-2\sqrt{3B}\right)>0, \\ E &:= \frac{4\gamma}{27}\left(y-g+2\sqrt{3B}\right)>0, \end{aligned} \tag{A12}$$

where $y := 6\left(s^2\gamma+\beta\right)$, and where the condition $y>2g$ ensures that these parameters (and therefore the possible solutions) are real. The positive sign of C is confirmed by noticing that

$$y-g>2\sqrt{3B}\Leftrightarrow (y-g)^2>\left(2\sqrt{3\left(\frac{1}{6}\left(\frac{y}{2}-g\right)y\right)}\right)^2, \tag{A13}$$

which always holds since

$$(y-g)^2-\left(2\sqrt{3\left(\frac{1}{6}\left(\frac{y}{2}-g\right)y\right)}\right)^2=g^2>0. \tag{A14}$$

There are six possible solutions to (A11), given by:

$$\alpha_3 = s\gamma - \frac{1}{9}A,\ \alpha_4 = \frac{2(6y+5g)}{9(6y-4g)}A,\ \alpha_5 = \frac{2}{9}A \tag{S1}$$

$$\alpha_3 = s\gamma + \frac{1}{9}A,\ \alpha_4 = -\frac{2(6y+5g)}{9(6y-4g)}A,\ \alpha_5 = -\frac{2}{9}A \tag{S2}$$

$$\alpha_3 = s\gamma - \left(\frac{6y-5g}{4g} - \frac{81}{16g\gamma}C\right)\sqrt{C}, \alpha_4 = \frac{1}{2}\sqrt{C}, \alpha_5 = -\frac{1}{2}\sqrt{C} \tag{S3}$$

$$\alpha_3 = s\gamma + \left(\frac{6y-5g}{4g} - \frac{81}{16g\gamma}C\right)\sqrt{C}, \alpha_4 = -\frac{1}{2}\sqrt{C}, \alpha_5 = \frac{1}{2}\sqrt{C} \tag{S4}$$

$$\alpha_3 = s\gamma - \left(\frac{6y-5g}{4g} - \frac{81}{16g\gamma}E\right)\sqrt{E}, \alpha_4 = \frac{1}{2}\sqrt{E}, \alpha_5 = -\frac{1}{2}\sqrt{E} \tag{S5}$$

$$\alpha_3 = s\gamma + \left(\frac{6y-5g}{4g} - \frac{81}{16g\gamma}E\right)\sqrt{E}, \alpha_4 = -\frac{1}{2}\sqrt{E}, \alpha_5 = \frac{1}{2}\sqrt{E} \tag{S6}$$

Global asymptotic stability requires $\alpha_3 < 0$, $\alpha_3+\alpha_5 < 0$ and $\alpha_3-\alpha_5 < 0$. We can immediately eliminate (S2) because $\alpha_3 > 0$. The same is true for (S1), since $\alpha_3+\alpha_5 = s\gamma+\frac{1}{9}A > 0$. Regarding (S4), notice that a sufficient condition for $\alpha_3 > 0$ is

$$\frac{6y-5g}{4g} - \frac{81}{16g\gamma}C = \frac{1}{4g}\left(3y-2g+6\sqrt{3B}\right) > 0, \tag{A15}$$

which always holds for $y > 2g$. Similarly, regarding (S6), a sufficient condition for $\alpha_3 > 0$ is

$$\frac{6y-5g}{4g} - \frac{81}{16g\gamma}E = \frac{1}{4g}\left(3y-2g-6\sqrt{3B}\right) > 0, \tag{A16}$$

which always holds since

$$(3y-2g)^2 - \left(6\sqrt{3B}\right)^2 = 2g(2g+3y) > 0. \tag{A17}$$

Thus, (S4) and (S6) can also be ruled out because $\alpha_3 > 0$. In the two remaining solutions – (S3) and (S5) – we have $\alpha_5 < 0$, implying that $\alpha_3 + \alpha_5 < \alpha_3 < \alpha_3 - \alpha_5$. For these two solutions, the conditions for global asymptotic stability therefore reduce to $\alpha_3 - \alpha_5 < 0$. For (S5) we have

$$\alpha_3 - \alpha_5 = s\gamma - \left(6y - 7g - \frac{81}{4\gamma}E\right)\frac{\sqrt{E}}{4g}, \tag{A18}$$

where

$$6y - 7g - \frac{81}{4\gamma}E = 3y - 4g - 6\sqrt{3B}. \tag{A19}$$

A necessary (but not sufficient) condition for $\alpha_3 - \alpha_5 < 0$ is

$$(3y - 4g)^2 - \left(6\sqrt{3B}\right)^2 = -2g\,(3y - 8g) < 0, \tag{A20}$$

which is violated for $y > \frac{8}{3}g$. Thus, the condition $y > \frac{8}{3}g$ is sufficient to rule out (S5).

Finally, for the only solution left, (S3), we have

$$\alpha_3 - \alpha_5 = s\gamma - \left(6y - 7g - \frac{81}{4\gamma}C\right)\frac{\sqrt{C}}{4g}. \tag{A21}$$

A necessary condition for $\alpha_3 - \alpha_5 < 0$ is therefore

$$6y - 7g - \frac{81}{4\gamma}C = 3y - 4g + 6\sqrt{3B} > 0, \tag{A22}$$

which holds for all $y > 2g$. It is straightforward to show that the second term in (A21) is monotonically increasing in y and decreasing in g (for $y > 2g$). Since y is monotonically increasing in β and g is monotonically decreasing in k and τ, and since the first term in (A21) does not depend on either of these parameters, if follows that (S3) satisfies the conditions for global asymptotic stability if β and/or τ are sufficiently large relative to k. In qualitative terms, this condition (β and/or τ sufficiently large relative to k) also ensures $y > \frac{8}{3}g$, which implies that (S3) is the unique globally asymptotically stable closed-loop solution.

In the steady state closed-loop solution we have

$$\begin{aligned} I_i &= \frac{\alpha_1 + \alpha_3 q_i + \alpha_5 q_j}{\gamma}, \\ I_i &= \delta q_i, \\ q^{CL} &= \frac{\alpha_1}{\gamma\delta - \alpha_3 - \alpha_5}, \end{aligned} \tag{A23}$$

where α_3 and α_5 are given by (S3). From the second and third line in (A9) we can define the following system of two equations in α_1 and α_2:

$$\begin{aligned}\alpha_1(\delta+\rho)-\frac{k}{3}-\frac{\alpha_1\alpha_3}{\gamma}-\frac{\alpha_2\alpha_5}{\gamma}-\frac{\alpha_1\alpha_5}{\gamma}&=0,\\ \alpha_2(\delta+\rho)+\frac{k}{3}-\frac{\alpha_2\alpha_3}{\gamma}-\frac{\alpha_1\alpha_4}{\gamma}-\frac{\alpha_1\alpha_5}{\gamma}&=0.\end{aligned} \tag{A24}$$

Solving this system yields the following solution for α_1:

$$\alpha_1=\frac{k\gamma\left(\alpha_3+\alpha_5-(\delta+\rho)\gamma\right)}{3\left((\delta+\rho)\left(2\gamma\alpha_3+\gamma\alpha_5-(\delta+\rho)\gamma^2\right)-\alpha_3^2+\alpha_5^2-\alpha_3\alpha_5+\alpha_4\alpha_5\right)}. \tag{A25}$$

From (S3), note that $\alpha_4=-\alpha_5$. We can therefore re-write α_1 as

$$\alpha_1=\frac{k\gamma\left(\alpha_3+\alpha_5-(\delta+\rho)\gamma\right)}{3\left(\left(\gamma(\delta+\rho)\left((2\alpha_3+\alpha_5)-(\delta+\rho)\gamma\right)\right)-\alpha_3(\alpha_5+\alpha_3)\right)}=\frac{k\gamma}{3\left(\gamma(\delta+\rho)-\alpha_3\right)}, \tag{A26}$$

so that

$$q^{CL}=\frac{\alpha_1}{\gamma\delta-\alpha_3-\alpha_5}=\frac{k\gamma}{3\left(\gamma(\delta+\rho)-\alpha_3\right)\left(\gamma\delta-\alpha_3-\alpha_5\right)}. \tag{A27}$$

Proof of Proposition 2

The closed-loop solution requires $g\leq\frac{y}{2}$. Since g is monotonically decreasing in τ while y does not depend on τ, this implies that the closed-loop solution exists for sufficiently high values of τ. At the lower limit of τ, implicitly given by $y=2g$, steady-state quality in the closed-loop solution is

$$q^{CL}\Big|_{y=2g}=\frac{k}{\left(\frac{2k^2}{3\tau}-\frac{\gamma\rho^2}{2}\right)+\frac{\rho}{4}\left(\sqrt{2\gamma\left(\frac{2k^2}{3\tau}-\frac{\gamma\rho^2}{2}\right)+(\gamma\rho)^2}-\gamma\rho\right)}, \tag{A28}$$

whereas steady-state quality in the open-loop solution is

$$q^{OL}\Big|_{y=2g}=\frac{k}{\frac{2k^2}{3\tau}-\frac{\gamma\rho^2}{2}}. \tag{A29}$$

A straightforward comparison of (A28) and (A29) shows that $q^{OL}\big|_{y=2g} > q^{CL}\big|_{y=2g}$ if

$$\sqrt{2\gamma\left(\frac{2k^2}{3\tau} - \frac{\gamma\rho^2}{2}\right) + (\gamma\rho)^2} - \gamma\rho > 0, \tag{A30}$$

or, equivalently,

$$\sqrt{\frac{2\gamma k}{q^{OL}\big|_{y=2g}} + (\gamma\rho)^2} - \gamma\rho > 0, \tag{A31}$$

which always holds. Since q^{OL} is independent of τ while q^{CL} is monotonically decreasing in τ, it follows that $q^{OL} > q^{CL}$ for all $g \leq \frac{y}{2}$. *Q.E.D.*

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