Optimization of Wastewater Treatment Processes

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Abstract

En este trabajo usamos el paquete de software LOQO para optimización de procesos de tratamiento de aguas residuales. Se minimizó una función costo, el volumen del reactor y el caudal de agua para diferentes valores de demanda química de oxígeno y nitrógeno en el efluente.

Keywords: Wastewater Process Design, Cost function minimization, LOQO.

1. Introduction

Wastewater, either domestic or industrial, treatment plants (WWTPs) are nowadays emerging everywhere, as authorities concerned with environment issues legislate tighter laws on water quality. The Activated Sludge system is by far the most widely used biological process in wastewater treatments and it usually consists of an aeration tank and a secondary settler tank. High costs associated with the construction and operation of this system, which threaten the very survival of many industries, require a wise optimization of the process. Even for operating plants, the optimization procedure seems crucial since operation costs are very high. In particular, to the activated sludge system, the cost associated with the aeration is the most predominant operation cost.

This paper is organized as follows. In Section 2 we describe the used aeration tank, its mass balances and the quality constraints. Section 3 presents the secondary settler and the corresponding mass balances. In Section 4 we explain and list the used objective functions, followed by the formulations of the optimization problems in Section 5. Finally, Sections 6 and 7 contains the computational experiences and the conclusions respectively.

2. The aeration tank

The aeration tank is the reactor where the biological reactions take place. Many mathematical models have been used to describe the processes in these reactors. We chose the activated sludge model n.1 (ASM1), described by Henze *et al.* [1], which considers both the elimination of the carbonaceous matter and the removal of the nitrogen compounds. This model is widely accepted by the scientific community, as it produces good predictive values by simulations. This means that all state variables keep their biological interpretation. The tank is a completely stirred tank reactor (CSTR) in steady state.

2.1. Mass balances

The mass balances were obtained using the Peterson matrix of the ASM1 model [1].

The generic equation for a mass balance around a certain system is

$$\text{In - Out + Reaction = Accumulation.}$$

In mathematical language, for a CSTR

$$\frac{Q}{V_a}\left(\xi_{in}-\xi\right)+r_i\left(\xi\right)=\frac{d\xi}{dt},$$

where Q is the wastewater flow to be treated, V_a is the aeration tank volume, ξ and ξ_{in} are the concentrations of the component around which the mass balances are being made inside the reactor and on entry, respectively. It is convenient to refer that in a CSTR the concentration of a compound is the same inside the reactor and at the effluent. The reaction term for the compound in question, r_i, is obtained by the sum of the product of the stoichiometric coefficients, ν_{ij}, with the expression of the process reaction rate, ρ_j, of the ASM1 Peterson matrix [1]

$$r_i=\sum_j \nu_{ij}\rho_j.$$

In steady state, the accumulation term given by $\frac{d\xi}{dt}$ is zero, because the concentration is constant in time. A WWTP in labor for a sufficiently long period of time without significant variations can be considered at steady state. As our purpose is to make cost predictions in a long term basis it is reasonable to do so.

The ASM1 model involves 8 processes incorporating 13 different components. The mass balances for the inert materials, S_I and X_I, are not considered because they are transport-only components. The measure unit adopted is $g\ COD/m^3$ and the equations obtained from the ASM1 model with mass balances are as follows. All the symbols used in these formulae and throughout the paper are listed in the Appendix - Notation.

Soluble substrate (S_S)

$$\begin{aligned} &\frac{-\mu_H}{Y_H}\frac{S_S}{K_S+S_S}\left(\frac{S_O}{K_{OH}+S_O}+\eta_g\frac{K_{OH}}{K_{OH}+S_O}\frac{S_{NO}}{K_{NO}+S_{NO}}\right)X_{BH} \\ &+k_h\frac{X_{BH}}{K_X X_{BH}+X_S}\left(\frac{S_O}{K_{OH}+S_O}+\eta_h\frac{K_{OH}}{K_{OH}+S_O}\frac{S_{NO}}{K_{NO}+S_{NO}}\right)X_S \\ &+\frac{Q}{V_a}\left(S_{S_{in}}-S_S\right)=0; \end{aligned} \tag{1}$$

Slowly biodegradable substrate (X_S)

$$\begin{aligned} &\left(1-f_p\right)b_H X_{BH}+\left(1-f_p\right)b_A X_{BA} \\ &-k_h\frac{X_{BH}}{K_X X_{BH}+X_S}\left(\frac{S_O}{K_{OH}+S_O}+\eta_h\frac{K_{OH}}{K_{OH}+S_O}\frac{S_{NO}}{K_{NO}+S_{NO}}\right)X_S \\ &+\frac{Q}{V_a}\left(X_{S_{in}}-X_S\right)=0; \end{aligned} \tag{2}$$

Heterotrophic active biomass (X_{BH})

$$\begin{aligned} &\mu_H\frac{S_S}{K_S+S_S}\left(\frac{S_O}{K_{OH}+S_O}+\eta_g\frac{K_{OH}}{K_{OH}+S_O}\frac{S_{NO}}{K_{NO}+S_{NO}}\right)X_{BH}-b_H X_{BH} \\ &+\frac{Q}{V_a}\left(X_{BH_{in}}-X_{BH}\right)=0; \end{aligned} \tag{3}$$

Autotrophic active biomass (X_{BA})

$$\mu_A\frac{S_{NH}}{K_{NH}+S_{NH}}\frac{S_O}{K_{OA}+S_O}X_{BA}-b_A X_{BA}+\frac{Q}{V_a}\left(X_{BA_{in}}-X_{BA}\right)=0; \tag{4}$$

Particulate products arising from biomass decay (X_P)

$$f_p b_H X_{BH}+f_p b_A X_{BA}+\frac{Q}{V_a}\left(X_{P_{in}}-X_P\right)=0; \tag{5}$$

Nitrate and nitrite nitrogen (S_{NO})

$$\begin{aligned} &-\frac{1-Y_H}{2.86Y_H}\mu_H\frac{S_S}{K_S+S_S}\frac{K_{OH}}{K_{OH}+S_O}\frac{S_{NO}}{K_{NO}+S_{NO}}\eta_g X_{BH} \\ &+\frac{\mu_A}{Y_A}\frac{S_{NH}}{K_{NH}+S_{NH}}\frac{S_O}{K_{OA}+S_O}X_{BA} \\ &+\frac{Q}{V_a}\left(S_{NO_{in}}-S_{NO}\right)=0; \end{aligned} \tag{6}$$

$NH_4^+ + NH_3$ nitrogen (S_{NH})

$$\begin{gathered}
-\mu_H \frac{S_S}{K_S + S_S} \left(\frac{S_O}{K_{OH} + S_O} + \eta_g \frac{K_{OH}}{K_{OH} + S_O} \frac{S_{NO}}{K_{NO} + S_{NO}} \right) i_{X_B} X_{BH} \\
-\mu_A \left(i_{X_B} + \frac{1}{Y_A} \right) \frac{S_{NH}}{K_{NH} + S_{NH}} \frac{S_O}{K_{OA} + S_O} X_{BA} + k_a S_{ND} X_{BH} \\
+\frac{Q}{V_a} \left(S_{NH_{in}} - S_{NH} \right) = 0; \qquad (7)
\end{gathered}$$

Soluble biodegradable organic nitrogen (S_{ND})

$$\begin{gathered}
-k_a X_{BH} S_{ND} + k_h \frac{X_{BH}}{K_X X_{BH} + X_S} \\
\times \left(\frac{S_O}{K_{OH} + S_O} + \eta_h \frac{K_{OH}}{K_{OH} + S_O} \frac{S_{NO}}{K_{NO} + S_{NO}} \right) X_{ND} \\
+\frac{Q}{V_a} \left(S_{ND_{in}} - S_{ND} \right) = 0; \qquad (8)
\end{gathered}$$

Particulate biodegradable organic nitrogen (X_{ND})

$$\begin{gathered}
b_H \left(i_{X_B} - f_p i_{X_P} \right) X_{BH} b_A \left(i_{XB} - f_p i_{XP} \right) X_{BA} \\
-k_h \frac{X_{BH}}{K_X X_{BH} + X_S} \left(\frac{S_O}{K_{OH} + S_O} + \eta_h \frac{K_{OH}}{K_{OH} + S_O} \frac{S_{NO}}{K_{NO} + S_{NO}} \right) X_{ND} \\
+\frac{Q}{V_a} \left(X_{ND_{in}} - X_{ND} \right) = 0; \qquad (9)
\end{gathered}$$

Alkalinity (S_{alk})

$$\begin{gathered}
-\mu_H \frac{S_S}{K_S + S_S} \left[\frac{i_{X_B}}{14} \frac{S_O}{K_{OH} + S_O} + \eta_g \left(\frac{1 - Y_H}{14 \times 2.86 Y_H} + \frac{i_{X_B}}{14} \right) \right. \\
\left. \times \frac{K_{OH}}{K_{OH} + S_O} \frac{S_{NO}}{K_{NO} + S_{NO}} \right] X_{BH} - \mu_A \left(\frac{i_{X_B}}{14} + \frac{1}{7 Y_A} \right) \\
\times \frac{S_{NH}}{K_{NH} + S_{NH}} \frac{S_O}{K_{OA} + S_O} X_{BA} + \frac{1}{14} k_a S_{ND} X_{BH} \\
+\frac{Q}{V_a} \left(S_{alk_{in}} - S_{alk} \right) = 0; \qquad (10)
\end{gathered}$$

Oxygen (S_O)

$$QS_{O_{in}} - QS_O + K_La\left(S_{O_{sat}} - S_O\right)V_a + \left(-\frac{1-Y_H}{Y_H}\mu_H\frac{S_S}{K_S+S_S}\right.$$
$$\left.\times\frac{S_O}{K_{OH}+S_O}X_{BH} - \frac{4.57-Y_A}{Y_A}\mu_A\frac{S_{NH}}{K_{NH}+S_{NH}}\frac{S_O}{K_{OA}+S_O}X_{BA}\right)V_a = 0. \tag{11}$$

For oxygen mass transfer, the aeration by diffusion is considered:

$$K_La = \frac{\alpha\ G_S\ \eta\ P_{O_2}\ 1333.3}{V_aS_{O_{sat}}}\theta^{(T-20)} \tag{12}$$

$$HenryO_2 = 708.0\ T + 25700.0. \tag{13}$$

2.2. Composite variables

In a real system, some state variables are, most of the time, not available for evaluation. Thus, readily measured composite variables are used instead. They are defined as follows.

Particulate COD

$$X = X_I + X_S + X_{BH} + X_{BA} + X_P; \tag{14}$$

Soluble COD

$$S = S_I + S_S; \tag{15}$$

Total COD

$$COD = X + S; \tag{16}$$

Volatile suspended solids

$$VSS = \frac{X}{icv}; \tag{17}$$

Total suspended solids

$$TSS = VSS + ISS; \tag{18}$$

Biochemical oxygen demand

$$BOD = f_{BOD}\left(S_S + X_S + X_{BH} + X_{BA}\right); \tag{19}$$

Total nitrogen of Kjeldahl

$$TKN = S_{NH} + S_{ND} + X_{ND} + i_{X_B}\left(X_{BH} + X_{BA}\right) + i_{X_P}\left(X_P + X_I\right); \tag{20}$$

Total nitrogen

$$N = TKN + S_{NO}. \tag{21}$$

2.3. Quality constraints

This kind of constraints are usually imposed by law. The most used are related with limits in the COD, nitrogen and solids at the effluent. In this paper we consider only limits in the soluble COD and in soluble nitrogen because our concern is on the performance of the aeration tank. In mathematical terms, these constraints are defined as:

$$S \leq S_{demanded} \tag{22}$$

$$N \leq N_{demanded}. \tag{23}$$

3. The secondary settler

Although the role of the secondary settler is sometimes underestimated, it plays a crucial part in the wastewater treatment. After the wastewater leaves the aeration tank, where the biological treatment took place, it is necessary to separate the biological sludge from the water. The most common way of achieving this is by using settling tanks. If this unitary process is not used, the COD at the effluent of the aeration tank would be larger than the one at the influent, due to the contribution of the biological sludge.

The most simple model of a secondary settler is obtained by assuming a simple separation point with perfect clarification [3]. If the design of the settling tank is the main goal then this model by itself is not of great use. However, it becomes highly important when the aim is to study the aeration tank that is adjacent in the activated sludge system (Figure 1). In its simplicity, it adds very important concepts as the recycle rate and the sludge retention time (SRT).

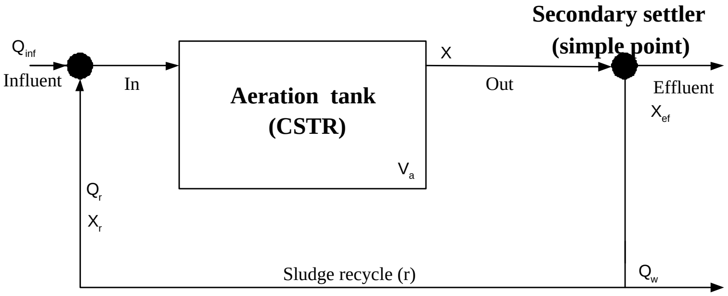

Figure 1: Schematic representation of the activated sludge system, considering the secondary settler as a simple separation point

The recycle rate is, by definition, the rate between the sludge flow that returns to the aeration tank and the influent flow that enters the system:

$$r = \frac{Q_r}{Q_{inf}}. \tag{24}$$

It is a very important concept in the correct maintenance of an aeration tank, being responsible for an adequate concentration of the sludge inside the reactor.

The SRT determines for how long the sludge is maintained inside the aeration tank. It is known in practice that for a good feature in the setting properties of the sludge, this parameter should range from 3 to 30 days. Its definition is:

$$SRT = \frac{V_a X}{Q_w X_r}. \tag{25}$$

Depending on the wastewater, the desirable SRT varies. For low polluted waters it is convenient a SRT of few days; for highly polluted waters the opposite is adequate. A high SRT implies a more concentrated sludge, a smaller aeration tank and a bigger sedimentation area for the settler tank. A prolonged aeration sludge with a STR between 20 and 30 days is considered. In terms of costs all of these factors should be taken into account.

The resulting mass balances to the particulate components, considering the system represented in Figure 1 are:

$$(1+r)\,Q_{inf}X_{S_{in}} = Q_{inf}X_{S_{inf}} + (1+r)\,Q_{inf}X_S - V_a\frac{X}{SRT}\left(\frac{X_{S_r}}{X_r}\right) \tag{26}$$

$$(1+r)\,Q_{inf}X_I = Q_{inf}X_{I_{inf}} + (1+r)\,Q_{inf}X_I - V_a\frac{X}{SRT}\left(\frac{X_{I_r}}{X_r}\right) \tag{27}$$

$$(1+r)\,Q_{inf}X_{BH_{in}} = Q_{inf}X_{BH_{inf}} + (1+r)\,Q_{inf}X_{BH} - V_a\frac{X}{SRT}\left(\frac{X_{BH_r}}{X_r}\right) \tag{28}$$

$$(1+r)\,Q_{inf}X_{BA_{in}} = Q_{inf}X_{BA_{inf}} + (1+r)\,Q_{inf}X_{BA} - V_a\frac{X}{SRT}\left(\frac{X_{BA_r}}{X_r}\right) \tag{29}$$

$$(1+r)\,Q_{inf}X_{P_{in}} = Q_{inf}X_{P_{inf}} + (1+r)\,Q_{inf}X_P - V_a\frac{X}{SRT}\left(\frac{X_{P_r}}{X_r}\right) \tag{30}$$

and for the dissolved matter, including oxygen, are:

$$(1+r)\,Q_{inf}S_{S_{in}} = Q_{inf}S_{S_{inf}} + rQ_{inf}S_{S_r} \tag{31}$$

$$(1+r)\,Q_{inf}S_{O_{in}} = Q_{inf}S_{O_{inf}} + rQ_{inf}S_{O_r} \tag{32}$$

$$(1+r)\,Q_{inf}S_{NO_{in}} = Q_{inf}S_{NO_{inf}} + rQ_{inf}S_{NO_r} \tag{33}$$

$$(1+r)\,Q_{inf}S_{NH_{in}} = Q_{inf}S_{NH_{inf}} + rQ_{inf}S_{NH_r} \tag{34}$$

$$(1+r)\,Q_{inf}S_{ND_{in}} = Q_{inf}S_{ND_{inf}} + rQ_{inf}S_{ND_r} \tag{35}$$

$$(1+r)\,Q_{inf}S_{alk_{in}} = Q_{inf}S_{alk_{inf}} + rQ_{inf}S_{alk_r} \tag{36}$$

$$QS = (Q - (Q_r + Q_w))\,S + (Q_r + Q_w)\,S_r. \tag{37}$$

4. The objective functions

In a preliminary study, simple objective functions are to be considered, namely the aeration tank volume, the air flow rate and the soluble COD. Then, an optimization problem is solved with a cost function.

4.1. Simple objective functions

These functions, although simple, allow us a detailed analysis of the system. When our concern is the investment cost, the aeration tank volume is to be minimized, as the construction of such a tank is its main contribution. The minimization of the air flow becomes more important when the operation costs are predominant. This acquires great importance when the WWTP is already operating. In this case, the aeration tank volume is considered as a parameter. A similar situation occurs when the soluble COD is to be minimized.

4.2. Cost functions

The cost function allows a much more detailed and reliable optimization of the system because it includes both investment and operation costs. In this paper, for the sake of simplicity, no pumps were considered, which means that all the flows in the system move by the effect of gravity.

The operation cost is usually on anual basis, so it has to be updated to a present value with the function Γ:

$$\Gamma = \sum_{j=1}^{n} \frac{1}{(1+i)^{j}} = \frac{1-(1+i)^{-n}}{i}, \tag{38}$$

where i is the discount rate and n is the life span of the WWTP. We use $i = 0.05$ and $n = 20$ years. The total cost is given by the sum of the investment (IC) and operation (OC) costs:

$$TC = IC + \Gamma OC. \tag{39}$$

To obtain a cost function based on portuguese real data, a study was carried out with a WWTP building company. The basic structure of the model is $C = aX^{b}$ [4], where a and b are the parameters to be estimated and X is the characteristic of the unit process that most influences the cost. This model

V	G_S	$HP\,(KW.h)$	$Cost(euro)$	
(m^3)	$(m^3/day\ STP)$	(anual basis)	Civil construction	Electromechanical
400	437.448	27720	87158.00	8416.00
600	596.52	129600	159796.00	10029.33
1000	942.5016	95040	189952.00	12198.67
1300	1282.518	162000	366145.50	17056.00

Table 1: Real data obtained for the aeration tank

is nonlinear in the parameters, but it can be easily linearized. The obtained linear model is

$$lnC = lna + b\ lnX$$

and the parameters lna and b are estimated by a least squares technique.

The real data collected from the portuguese company is presented in Table 1 and the investment cost function obtained is

$$IC_a = 148.6V_a^{1.07} + 7737G_S^{0.62}. \tag{40}$$

The collected information comes from a set of WWTPs in design, thus operation data are not available. However, from the company experience, the maintenance expenses for the civil construction are around 1% of the investment costs during the first 10 years and around 2% otherwise. For the electromechanical components, the maintenance expenses are negligible, but all the materials are usually replaced after 10 years. With this information and with the function Γ (38), the operation cost of the aeration tank is then

$$OC_a = \left[0.01\Gamma + 0.02\Gamma\,(1+i)^{-10}\right]\left(148.6V^{1.07}\right) + (1+i)^{-10}7737G_S^{0.62}. \tag{41}$$

The term $(1+i)^{-10}$ is used to bring to present a future value, in this case, 10 years from now.

5. Problem formulations

We have been formulating different optimization problems depending on the considered WWTP model. In a first stage, only the aeration tank is taken into account. In this situation, we minimize either the aeration tank volume (V_a), the air flow (G_s) or the chemical oxygen demand (COD).

The constraints obtained from the processes occurring in the aeration tank are described by equations (1) - (13). The composite variables (14) - (21) and the quality constraints (22) and (23) are also considered. All variables must be nonnegative, although more restricted bounds are imposed to some of the variables due to operational consistencies, namely:

$$0 \leq K_L a \leq 300$$

$$0.05 \leq HRT \leq 2$$

$$20 \leq SRT \leq 30$$

$$800 \leq TSS \leq 6000$$

$$2500 \leq TSS_r \leq 10000$$

$$0.5 \leq r \leq 2$$

$$6 \leq S_{alk} \leq 8$$

$$6 \leq S_{alk_{in}} \leq 8$$

$$S_O \geq 2.$$

In the next stage, a secondary settler is considered with the model described in Section 3. Equations (24) and (25), and the constraints corresponding to the mass balances (26) - (37) are added to the formulation. In this problem, the cost function (39), (40) and (41) is minimized. The dissolved oxygen was set to 2, which is usual in practice.

6. Computational experiences

The optimization problems were coded in the modelling language AMPL [5] and were solved using the well-known software package LOQO [2]. The parameters used are shown in Tables 2 to 4.

Parameter	Value	Parameter	Value
Y_A	0.24	i_{X_B}	0.086
Y_H	0.666	i_{X_P}	0.06
f_P	0.08		

Table 2: Stoichiometric parameters

For an easy interpretation of the results some graphs are presented in which a quality index (QI) [6] that defines the amount of pollution at the effluent appears:

$$QI = (COD + 2 \times BOD) \frac{Q_{ef}}{1000}$$

where the considered COD and BOD correspond to the soluble fraction.

Parameter	Value	Parameter	Value
μ_H	6	k_h	3
K_S	20	K_X	0.03
K_{OH}	0.2	μ_A	0.8
K_{NO}	0.5	K_{NH}	1
b_H	0.62	b_A	0.04
η_g	0.8	K_{OA}	0.4
η_h	0.4	k_a	0.08

Table 3: Kinetic parameters

Parameter	Value	Parameter	Value
T	20	α	0.8
P_{O_2}	0.21	η	0.07

Table 4: Operational parameters

6.1. Results for the set of problems considering only the aeration tank

The characteristics of the influent to the aeration tank are presented in Table 5.

It can be seen from Tables 6 and 7 that the soluble COD (S) attains always the maximum value allowed, except for the last case ($S \leq 50$). For limits of 50 g/m^3 and higher, the soluble COD at the effluent never rises above 45.2 g/m^3. This is due to the lower bound imposed on the HRT of 0.05 $days$, which implies that the volume of the aeration tank (V_a) has a minimum of 200 m^3. For this tank, the minimum treatment achieved is the value of $S = 45.2 g/m^3$.

Parameter	Value	Parameter	Value
Q	4000	$X_{I,in}$	727.3
$S_{I,in}$	30	$X_{S,in}$	422.4
$X_{BH,in}$	355.5	$S_{NH,in}$	9.54
$X_{BA,in}$	9.49×10^{-7}	$S_{ND,in}$	0.6129
$X_{P,in}$	87.06	$X_{ND,in}$	17.39
$S_{O,in}$	0	$X_{II,in}$	588.6
$S_{NO,in}$	5×10^{-7}	X_{in}	1592.26
$S_{alk,in}$	6.846	S_{in}	62.78
$S_{S,in}$	32.78		

Table 5: Characteristics of the wastewater at the influent to the aeration tank

When the objective function is the aeration tank volume, for the same quality index at the effluent, the total cost is always higher than in the case where the air flow is minimized (see Figure 2). This means that the operation cost has a more significant contribution to the total cost than the investment cost.

S limit	35	40	45	50
Iterations	427	66	79	91
V_a	2788	918	207.5	200
K_La	300	300	300	265.6
G_S	216 316	71 226	16 103	13 740
S	35	40	45	45.2
$N_{soluble}$	8.5	5.9	8.3	8.4
HRT	0.70	0.23	0.052	0.05

Table 6: Results considering the aeration tank volume as objective function, for $N_{soluble} \leq 15 g/m^3$

S limit	35	40	45	50
Iterations	135	54	139	127
V_a	2947	939	211	200
K_La	146.2	223.5	241.4	243.8
G_S	111 413	54 301	13 143	12 612
S	35	40	45	45.2
$N_{soluble}$	8.8	6.0	8.3	8.4
HRT	0.74	0.23	0.053	0.05

Table 7: Results considering the air flow as objective function, for $N_{soluble} \leq 15 g/m^3$

In Table 8 the volume is considered as a parameter, which simulates a established WWTP. The soluble COD at the effluent is always under the limit imposed of 45 g/m^3. In this case, the tank volume is limiting the treatment. As we are minimizing the air flow for a fixed value of the aeration tank volume, the dissolved oxygen is kept at 2 g/m^3, that corresponds to the minimum concentration allowed into the reactor. This level of oxygen implies a quality at the effluent superior to the minimum demanded, and for this reason the quality (S) improves as the volume increases, always under the imposed lower bound.

When soluble COD is the objective function (Table 9), two distinct situations are considered. In the first, the volume is set as a variable (second column of Table 9) and the convergence was very slow. In real life, this situation

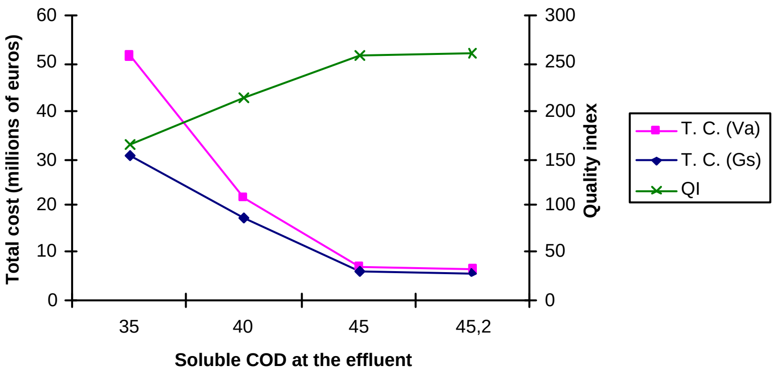

Figure 2: Total cost and quality index *versus* soluble COD at the effluent, for the minimization of the aeration tank volume and the air flow

V_a	500	1000	1500	2000	2500
$Iterations$	20	25	26	26	26
$K_L a$	218.3	224.4	214.5	186.3	162.4
G_S	28 227	58 055	83 212	96 382	105 043
S	41.6	39.8	38.1	36.6	35.6
$N_{soluble}$	7.1	5.9	6.0	6.9	7.9

Table 8: Results considering the air flow as objective function and the aeration tank volume as a parameter, for $N_{soluble} \leq 15g/m^3$ and $S \leq 45g/m^3$

does not make much sense since it means that there is no concerning with the costs. The volume obtained in this case is very high (9892 m^3) and besides operational problems, it would lead to an incomportable cost. In the second situation, the volume is set as a parameter, meaning that the performance of a WWTP already operating will be optimized. From Figure 3, the optimization of the quality of the effluent (S) is not compensative. Despite the cost is much higher than when the objective function is the air flow, there is no significative gain in the effluent quality, measured by the quality index.

Another observation from the Tables 6 to 9 is that the soluble nitrogen ($N_{soluble}$), with exception of the second column of Table 9, is always under the limit of 15 g/m^3. As the soluble nitrogen is a small fraction of the total nitrogen, this is not unusual. On the other hand, the limiting quality constraint for the project of a WWTP is the soluble COD. The limit in the soluble nitrogen is only achieved when precisely the soluble COD is being optimized. In this case,

V_a	$Variable$	500	1000	1500	2000	2500
$Iterations$	9892	208	369	3234	5898	7675
V_a	6488	500	1000	1500	2000	2500
K_La	143.1	300	300	300	300	300
G_S	240 179	38 799	77 599	116 298	155 197	193 997
S	33.1	41.6	39.7	37.9	36.4	35.4
$N_{soluble}$	15.0	7.1	5.8	6.0	6.8	8.2
HRT	1.62	0.125	0.25	0.375	0.5	0.625

Table 9: Results considering the soluble COD as objective function and the aeration tank volume first as variable and then as a parameter, for $N_{soluble} \leq 15 g/m^3$ and $S \leq 45 g/m^3$

the system will neglect, as much as possible, the quality in terms of nitrogen to improve it in terms of carbonaceous matter (COD).

6.2. Results for the problem considering the full activated sludge system

The characteristics of the influent to the system are presented in Table 10.

Parameter	Value	Parameter	Value
Q_{inf}	2000	$X_{I,inf}$	73.65
$S_{I,inf}$	30	$X_{S,inf}$	123
$X_{BH,inf}$	0	$S_{NH,inf}$	11.7
$X_{BA,inf}$	0	$S_{ND,inf}$	0.63
$X_{P,inf}$	0	$X_{ND,inf}$	1.251
$S_{O,inf}$	0	$X_{II,inf}$	59.6
$S_{NO,inf}$	0	X_{inf}	196.7
$S_{alk,inf}$	7	S_{inf}	82.73
$S_{S,inf}$	52.73		

Table 10: Characteristics of the influent to the system

When the full activated sludge system is considered (Table 11 and Figure 4), the optimization was carried out based on the minimization of the cost function. As expected, the cost decreases as the quality index deteriorates. The aeration tank volume does not vary too much (the values are between 1383 m^3 and 1520 m^3), but on the other hand, the air flow decreases drastically from a value of 12479 $m^3/day\ STP$ to 754.6 $m^3/day\ STP$, which confirms the fact that the operation costs are predominant when the life span of the WWTP is considered.

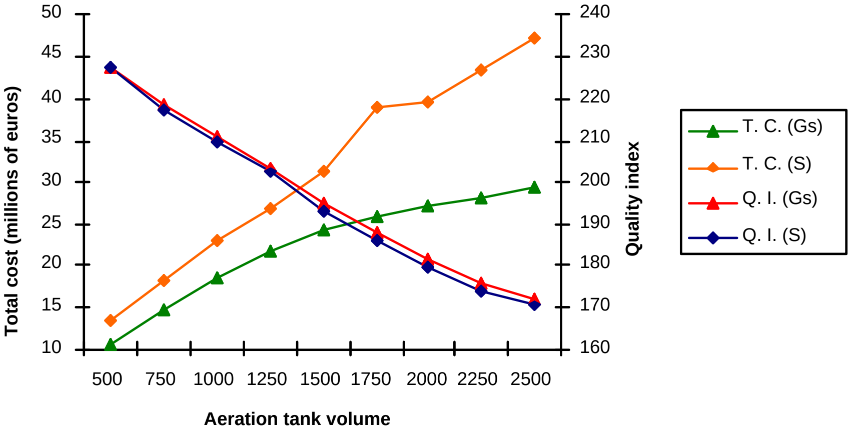

Figure 3: Total cost and quality index *versus* aeration tank volume, for the minimization of the air flow and the soluble COD at the effluent

The soluble COD at the effluent reaches, once again, the limit imposed, except when the value of the limit equals the one at the influent to the aeration tank ($S = 82.73$).

The sludge retention time (SRT) is always the considered lower bound, 20 $days$, because this implies a smaller volume to the aerator and a lower air flow.

Other experiences with different values of soluble nitrogen ($N_{soluble}$) concentration limit were carried out, but no changes in the total cost were observed. The reason for this behavior is that only the soluble nitrogen is considered and this is a small fraction of the total nitrogen.

7. Conclusions

We may conclude that the quality directly influences the cost of a WWTP project, especially in terms of carbonaceous matter. As higher quality to the effluent is demanded, the total cost increases.

When an operating WWTP is considered, there is no much point in optimizing the effluent quality (soluble COD) as the total cost gets much higher than the one obtained when the air flow is optimized for a similar quality index.

Another important conclusion is that when the life span of a WWTP is considered for a design, the operation cost is predominant over the investment

S *limit*	45	55	65	75	85
Iterations	78	89	94	105	155
Total cost	6.22	3.98	3.06	2.30	1.28
V_a	1383	1520	1520	1498	1476
G_S	12479	6146	3991	2426	754.6
K_La	34.9	15.6	10.2	6.26	1.98
r	0.50	1.01	1.36	1.52	0.5
HRT	0.46	0.38	0.32	0.30	0.49
SRT	20.0	20.0	20.0	20.0	20.0
Q_w	133	165	176	174	170
S_{ef}	45.0	55.0	65.0	75.0	82.73

Table 11: Results for the design and operational variables, considering the cost function as objective function, for $N_{soluble} \leq 15g/m^3$

cost.

8. Acknowledgement

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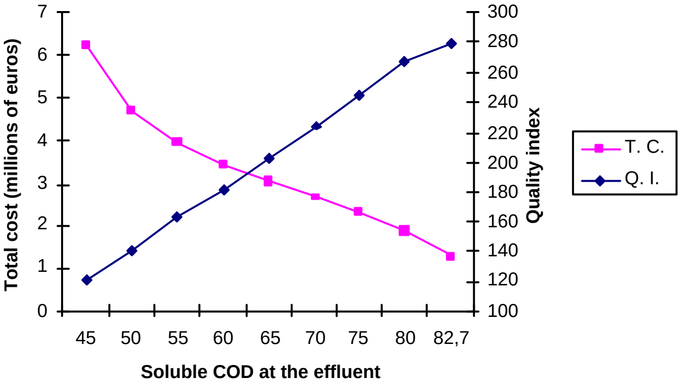

Figure 4: Total cost and quality index *versus* soluble COD at the effluent, for the minimization of the cost function, considering the full activated sludge system

Appendix - Notation

The following symbols are used in this paper:

b_A=decay coefficient for autotrophic biomass, day^{-1}

b_H=decay coefficient for heterotrophic biomass, day^{-1}

BOD=biochemical oxygen demand, $g\ O_2/m^3$

BOD_U=ultimate BOD, $g\ O_2/m^3$

COD=chemical oxygen demand, $g\ COD/m^3$

f_{BOD}=$BOD : BOD_U$ ratio

f_P=fraction of biomass leading to particulate products

G_S=air flow rate, m^3/day at STP

$HenryO_2$=Henry constant

HRT=hydraulic retention time, day

i=discount rate

icv=$X : VSS$ ratio, $g\ COD/g\ VSS$

i_{X_B}=nitrogen content of active biomass, $g\ N/g\ COD$

i_{X_P}=nitrogen content of endogenous/inert biomass, $g\ N/g\ COD$

IC=investment cost, $2003euros$

ISS=inorganic suspended solids, g/m^3

k_a=ammonification rate, $m^3/g\ COD/day$

k_h=maximum specific hydrolysis rate, day^{-1}

K_La=overal mass transfer coefficient, day^{-1}

K_{NH}=ammonia half-saturation coefficient for autotrophic biomass growth, $g\ N/m^3$

K_{NO}=nitrate half saturation coefficient for denitrifying heterotrophic biomass, $g\ N/m^3$

K_{OA}=oxygen half-saturation coefficient for autotrophs growth,$g\ O_2/m^3$

K_{OH}=oxygen half-saturation coefficient for heterotrophs growth, $g\ O_2/m^3$

K_S=readily biodegradable substrate half-saturation coefficient for heterotrophic biomass, $g\ COD/m^3$

K_X=half-saturation coefficient for hydrolysis of slowly biodegradable substrate, $g\ COD/g\ COD$

n=life span of the treatment plant, $years$

N=total nitrogen, $g\ N/m^3$

OC=operation costs, $2003euros$

P_{O_2}=partial pressure of oxygen uncorrected, i.e. 0.21

Q=flow, m^3/day

QI=quality index, Kg of $pollution/day$

r=recycle rate

S=soluble COD, $g\ COD/m^3$

S_{alk}=alkalinity, molar units

S_I=soluble inert organic matter, $g\ COD/m^3$

S_{ND}=soluble biodegradable organic nitrogen, $g\ N/m^3$

S_{NH}=free and ionized ammonia, $g\ N/m^3$

S_{NO}=nitrate and nitrite nitrogen, $g\ N/m^3$

S_O=dissolved oxygen, $g\ (-COD)/m^3$

$S_{O_{sat}}$=saturated oxygen concentration, g/m^3

S_S=readily biodegradable soluble substrate, $g\ COD/m^3$

SRT=sludge retention time, day

STP=standard temperature and pressure

TC=total costs, $2003euros$

V_a=aeration tank volume, m^3

VSS=volatile Suspended Solids, g/m^3

T=temperature, ^oC

TKN=total nitrogen of Kjeldahl, $g\ N/m^3$

TSS=total Suspended Solids, g/m^3

X=particulate COD, $g\ COD/m^3$

X_{BA}=active autotrophic biomass, $g\ COD/m^3$

X_{BH}=active heterotrophic biomass, $g\ COD/m^3$

X_I=particulate inert organic matter, $g\ COD/m^3$

X_{II}=inert inorganic suspended solids, g/m^3

X_{ND}=particulate biodegradable organic nitrogen, $g\ N/m^3$

X_P= particulate products arising from biomass decay, $g\ COD/m^3$

X_S=slowly biodegradable substrate, $g\ COD/m^3$

Y_A=yield for autotrophic biomass, $g\ COD/g\ N$

Y_H=yield for heterotrophic biomass, $g\ COD/g\ COD$

α=wastewater/clean water coefficient

η=standard oxygen transfer efficiency

η_g=correction factor for μ_H under anoxic conditions

η_h=correction factor for hydrolysis under anoxic conditions

μ_A=maximum specific growth rate for autotrophic biomass, day^{-1}

μ_H=maximum specific growth rate for heterotrophic biomass, day^{-1}

θ=temperature correction factor

subscripts

a=aeration tank

in=entery of the aeration tank

inf=influent

r=recycle

w=sludge waste