Bayesian approach to fuse NDT data to find Dynamic Elastic Modulus of granite Stone

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ABSTRACT:
The main objective of the paper is to propose a criterion for combining NDT test data coming from sonic and ultrasonic tests to evaluate dynamic elastic modulus of granite stone. Bayesian updating allows us to combine initial information with new information to provide more realistic results. The paper fuses NDT test data gathered from sonic and ultrasonic tests having uncertainties in velocity, Poisson ratio and density values of granite stone under consideration using Bayesian statistics into a normal probabilistic distribution plot of dynamic elastic modulus. Many times especially in case of heritage buildings destructive tests cannot be performed because of minimum intrusion guidelines, so one has to rely on NDT test data to gain knowledge about the mechanical parameters of the building.

Keywords: Dynamic elastic modulus, Bayesian, NDT, Uncertainty, Sonic tests

NOTATION
PDF Probability density function;  
$E_d$ Dynamic Elastic Modulus;  
NDT Non-Destructive testing;  
$\nu$ Poisson Ratio;  
$\nu_p$ Velocity of p wave;  
$\rho$ Density of granite stone.

1 INTRODUCTION
In the case of monuments and historical constructions it is not possible to carry out a scheme of testing which involves intensive use of invasive techniques by taking out samples from the structure also mentioned in guidelines laid by ICOMOS [1]. Since, no historical data regarding characterization of granite used for St Torcato church was available so to analyse the structure mechanical characteristics of the material must either be determined or estimated. To gain information about the structure only NDT seems a viable option for inspection and deductions need to be made from the test data coming from these tests. Data fusion (data integration) sometimes also known as information fusion is the process of merging data coming from different sources into a representation that provides effective support in decision making [2]. The main role of data fusion through several

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publications [3-5] is its ability to manage uncertainties and improve accuracy of the system. Data fusion in case of NDT is still a new concept and needs to be understood to be practised by engineers. Flowchart shown in Figure 1 shows the concept of data fusion adapted from [4] to fuse NDT data from sonic and ultrasonic tests obtained from NDT testing of St. Torcato Church located in a small village near Guimarães to arrive to a dynamic elastic modulus value of granite block.

![Flowchart](image)

**Figure 1**: Illustration of data fusion system used for St Torcato Church

Most of the data fusion applications are limited to multisensor integration of information and integration of NDT test data of geomechanical parameters is a new area. Many data fusion algorithms can be used in the fusion operator to fuse pieces of data, but we have considered only Bayesian Inference [6-7] in this case. The number of fusion processes can be limited by the number of data sources and one can apply fusion operator each time a new piece of data is obtained. Some research papers using Bayesian techniques related to geomechanical parameters include:

1. Assessment of bridges using Bayesian updating in which yield strength of rebars and concrete cover was updated from NDT test data [5].


4. Bayesian methodology to update elastic modulus and quantifying uncertainties present in it [3].

## 2 DATA COLLECTION

Data collection was done by reading and going through several testing reports [10] of St Torcato church and also going through several excel data sheets available from the testing scheme. Two types of data were collected in this case: one was experimental data (direct and indirect) obtained from testing and another type was monitoring data obtained from continuous monitoring of St. Torcato church related to its structural condition and conservation. But only the experimental data from sonic/ultrasonic tests was used as a case study in this paper and the monitoring data was discarded. In data collection, two granite blocks were taken from the quarry site to the lab and tested. We had the information that the blocks came from the same quarry, but they used two quarries to build the church. So, there is uncertainty related with this aspect. The two blocks taken to the laboratory for testing gave similar values for Schmidt hammer test with in-situ testing of granite block from St Torcato church. Both sonic and ultrasonic test can give qualitative information about voids present
inside masonry and can be correlated to some mechanical parameters providing broader information about the structure. The determination of mechanical parameters is of paramount importance as they provide information about the behaviour of masonry material required for further study of this church.

![Figure 2. a) Sonic test setup: Marking of locations b) Locations P1 through P5](image)

### 2.1. GRANITE SONIC TESTS

The sonic tests were done on points on northwest wall of west tower (See Figure ) and the same comparison was made with the granite blocks. Some values shown in Table 1 are a bit low which shows discontinuity in rubble masonry and presence of voids. The same sample taken from the quarry was used to perform three types of tests: direct, semi-direct and indirect. The velocity can be used as an indicator to get an idea about the voids present inside the wall. For carrying out the data fusion process only results of direct sonic tests performed in-situ were taken into account since they show less coefficient of variation than the other indirect and semi-direct tests.

#### Table 1. Table showing results of direct sonic tests from St. Torcato church [10]

<table>
<thead>
<tr>
<th>S.No</th>
<th>Location</th>
<th>Avg. Velocity (m/s)</th>
<th>COV</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>P1</td>
<td>2073</td>
<td>1.84</td>
</tr>
<tr>
<td>2</td>
<td>P2</td>
<td>4220</td>
<td>4.54</td>
</tr>
<tr>
<td>3</td>
<td>P3</td>
<td>3244</td>
<td>2.39</td>
</tr>
<tr>
<td>4</td>
<td>P4</td>
<td>3821</td>
<td>3.38</td>
</tr>
<tr>
<td>5</td>
<td>P5</td>
<td>3322</td>
<td>2.39</td>
</tr>
</tbody>
</table>

### 2.2. GRANITE ULTRASONIC TESTS

The same kind of tests which were performed on sonic (See section 2.1) was repeated for ultrasonic range details of which are explained in Report no 6 [10]. The direct sonic test count not be performed on the granite at the church because there were no obvious locations where the granite went all the
way through the wall without any voids or joints. Also, these tests were not good since the analysis of signal was done internally making it impossible to know which wave form was used in determining the results. The other test data with high coefficient of variation (COV) was discarded due to more variability and less reliability. The same argument can be used as the tests can also incorporated into the numerical model with additional uncertainty making the model more complex. For ultrasonic testing in-situ tests were not done so the data which came from two granite blocks 1 and 2 was used and locations of these tests is shown in Figure 3. Table 2 summarizes the data obtained from stone blocks 1 and 2.

![Figure 3. Granite block ultrasonic test locations for stone 1 and stone 2](image)

<table>
<thead>
<tr>
<th>Surfaces</th>
<th>Block 1 velocity</th>
<th>Block 2 velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>m/s</td>
<td>m/s</td>
</tr>
<tr>
<td>A-C</td>
<td>3836</td>
<td>3694</td>
</tr>
<tr>
<td>D-E</td>
<td>3885</td>
<td>3849</td>
</tr>
</tbody>
</table>

### 3 BAYESIAN TECHNIQUE OF COMBINATION

It is reasonable to assume prior as a normal distribution for modelling of geomechanical parameter also verified by a study conducted by LNEC [11]. The values for population of $E_d=f(v_p,v_n,\rho)$ [Table 3] were generated using Monte Carlo simulation methods. A total of 10000 values were generated by writing a simple Matlab code using Equation 1 in which velocity, Poisson ratio and density varying randomly over a specified range and the mean and standard deviation of population was reported to be used for the fusion process. So, to calculate dynamic elastic modulus (based on vibration and wave propagation) Equation 1 was used.

$$E_d = v_p^2 \frac{(1+v)(1-2v)}{(1-v)} \rho$$  \hspace{1cm} (1)

![Table 3. Range of parameter values having uncertainty for granite](image)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_{ultrasonic}$ [NDT test data]</td>
<td>3694-3885</td>
</tr>
<tr>
<td>$v_{sonic}$ [NDT test data]</td>
<td>2073-4220</td>
</tr>
<tr>
<td>$\nu$ [Engineering Toolbox]</td>
<td>0.2-0.3</td>
</tr>
<tr>
<td>$\rho$ (kg/m3) [Engineering Toolbox]</td>
<td>2600-2800</td>
</tr>
</tbody>
</table>

After this first step of statistical processing of data, the values of mean and standard deviation from these two simulations (Refer Table 4) are fed into the fusion center. We have used simulation evaluation of velocity by two indicative tests: sonic and ultrasonic tests. The idea behind the whole process is to arrive to a posterior distribution using likelihood of the data. The parameters of interest (mean and standard deviation of dynamic elastic modulus) are considered to be random variable with
variable moments which means that mean and variance are random variable following a certain distribution. For details of the algorithm and mathematical formulation to model uncertainties on uncertainties and of the fusion operator used paper of Miranda et al.[3] can be referred for details. The proposed model deals with many kinds of material uncertainties represented by means of probability theory [12]. In most cases, the NDT data with low reliability shows more standard deviation and data with high reliability shows less standard deviation.

| Table 4. Values of dynamic elastic modulus generated using Monte Carlo simulation |
|------------------|-----------------|-----------------|
| Parameter       | Sonic           | Ultrasonic      |
| \( E_d \)       | 23.1            | 32.18           |
| \( \sigma \)    | 8.87            | 2.08            |

![Figure 4](image)

Figure 4. Fused Posterior density function of dynamic elastic modulus \( E_d \) using conjugate prior

### 4 RESULTS OF DATA FUSION

The Figure 4 shows the resulting elastic modulus mean and standard deviation values in form of fused posterior distribution. While carrying out data fusion equal weightage of both sonic and ultrasonic test was taken in the algorithm used for fusion. Table 5 can be referred for summarizing all the results. Distribution is in good agreement with test data as can be seen from result in which mean and standard deviation lie in between the values obtained from these two tests.

| Table 5. Posterior estimates of \( E_d \) in GPa using conjugate prior |
|------------------|----------------|
| Parameter        | Conjugate      |
| \( \mu \)        | 27.64          |
\[ \sigma(\mu) = 1.01 \]
\[ \sigma = 6.36 \]
\[ \sigma(\sigma) = 0.72 \]
\[ 95\% \text{ confidence for mean} = 25.98-29.31 \]
\[ \mu_{\text{pop}} = 27.65 \]
\[ \sigma_{\text{pop}} = 7.41 \]
\[ 95\% \text{ confidence for population mean} = 15.45-39.84 \]

5 CONCLUSIONS

The new application of data fusion presented in this paper will help practitioners working with NDT data who want to choose parameters based on the results of different NDT tests having different levels of reliability and uncertainty quantification. The paper presents the final parameter in form of a normal distribution and also in numerical format easily understood by people working with NDT data. The Matlab toolbox which was developed in the case study for NDT data fusion for St Torcato church was used for this data set (See snapshot of graphical user interface of toolbox in Figure 5). The paper uses data from sonic/ultrasonic tests but the approach can be adapted to different data sets. The value of static elastic modulus (E) can be obtained by using the appropriate conversion factor for granite stone. Using Bayesian approach to combine NDT test data to evaluate dynamic elastic modulus (E_d) and dealing with certain levels of uncertainty can be considered as main contribution of this paper.

![Figure 5. Calculation of E_d using toolbox developed](image)

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