Using Fuzzy Logic to Characterize Uncertainty during the Design and Use Stages of Performance Measurement

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Abstract—The process of performance measurement encompasses the activities required for data collection (use stage), which was previously designed (design stage) and contribute to decision-making after data analysis (analysis stage). The lack of quality of performance measures (PMs) may influence decision-making. Since the process of performance measurement involves generally several actors, the decision-maker may not be aware of the level of uncertainty associated with performance measures. In this paper, fuzzy sets are used to represent the uncertainty generated in performance measures during its design, use and analysis stages. The uncertainty sources are arranged on three cause–and-effect diagrams representing controllable factors that can lead to imperfect design, use and analysis, impacting on PMs uncertainty. This degree of imperfection will be labelled deficiency (at a given stage) and a methodology is presented to infer its effect on the PM uncertainty. The identification of uncertainty sources and the determination of an Uncertainty Index support actions to improve performance measures’ quality. An application example is provided to show the usefulness of the proposed methodology.

Index Terms—Data Quality, Decision-making, Fuzzy sets, Performance Measures, Uncertainty.

I. INTRODUCTION

The monitoring of efficiency and effectiveness of processes or systems allows managers to control, decide, implement and observe the effects of their actions to understand if they are moving towards achieving their goals. The definition and selection of performance measures (PMs) have attracted the attention of several authors and different knowledge areas propose extensive lists of PMs. Models, framework and standards are available in literature for the selection of an adequate set of PMs for a given process or business.

PMs can be considered a particular type of Data or Information and the literature refers some dimensions or attributes of Data/Information Quality as [1]: accuracy; completeness; timeliness; and consistency. This implicitly suggests that all data may lack some of these attributes.

Several other classifications for Data/Information Quality are proposed in literature [2]. For example, Galway and Hanks [3] classify data quality problems as operational, conceptual and organizational. The quantification of uncertainty can contribute to highlight the lack of accuracy and precision of data or PMs. The decision-maker should know the existence and magnitude of Uncertainty on PMs, once this uncertainty may lead to higher risk in decisions. The study of the sources of PMs’ uncertainty can also contribute to identify adequate initiatives to improve the process of designing, using and analyzing the PMs. In this context, in order to get useful information about factors that induce uncertainty in PMs, adequate modeling tools can be applied, as is the case of fuzzy logic.

Fuzzy logic was introduced by Zadeh [4] as a mathematical technique to represent vagueness in everyday life. Fuzzy logic methodology can model complicated processes and deal with qualitative imprecise or vague knowledge and information [5]. It provides a tool for directly working with the linguistic terms used in the assessment of factors that contribute to uncertainty of PMs, and has many applications in Performance measurement field [6-9]. When the available information from the process is qualitative, inexact, incomplete, imprecise, vague or uncertain, the notion of the membership function utilized by fuzzy theory is then adequate for depicting this knowledge.

To contribute to the field of Performance Measurement and to the field of Data Quality, this work studies the causes of uncertainty on the process of using PMs. Fuzzy numbers are used to develop a methodology that aims to obtain the value of an index for uncertainty evaluation of a given PM or (key) performance indicator.

The paper is organized as follows. Section 2 presents a literature review about performance measurement. In Section 3, the stages of the performance measurement process and the uncertainty sources that may affect it in each stage are defined. Section 4 presents the proposed methodology based on fuzzy logic to estimate PM uncertainty. Section 5 and section 6 present, respectively, an example of application of the methodology and the final conclusion of this work.

II. PERFORMANCE MEASUREMENT

Performance measurement can be described as a set of steps involving firstly the design and implementation of the performance measure and, secondly, the use and review of performance measures. Concerning the first two steps,
design and implementation, there are several works in literature that propose and define different performance measures in different knowledge area and that discuss the implementation difficulties and critical success factors [10]. The “use” step involves a sequence of tasks that are repeated each time a new value of the performance measure is requested. These tasks aim to collect the necessary data and present results [11].

Juran and Godfrey [11] argue “the choice of what to measure and the analysis, synthesis, and presentation of the information are just as important as the act of measurement itself”. These authors also argue that the measurement process belongs to a larger measurement system, which embraces the decisions that are made and the framework in which the process operates. This large system involves, typically, different actors.

A set of related PMs can be called a performance measurement system (PMS). Many PMSs are available to companies, such as the Balanced Scorecard, but many other activities related with assessing processes’ performance or comparing the performance of similar subsystems, or generally doing benchmarking exercises, all rely on (key) PMs. To design a Performance Measurement System it is crucial to understand who will make the decisions (and how) and who will take actions [11], i.e. the purpose of each PM must be clear [12], and must promote the company’s strategy [13]. Before determining what to measure and how to measure it, the overall framework in which the Performance Measurement System operates should be understood [11]. It can be concluded that the relevance of PMs is related to decisions they can support and that there are no bad PMs, only the bad use of them [14].

To increase quality of PMs some of its attributes or requirements are identified in the literature [13-15]: relevant; credible; precise; valid; reliable and frequent. Other authors refer some recommendations for both the performance measurement process and performance measures:

- data collection and methods for calculating the PMs must be clearly defined [16];
- presentation of PMs must be simple [17];
- PMs must be flexible [15], including being tied to desired results [18];
- more extensive use should be made of subjective data [13]; and
- ratio-based performance criteria are preferred to absolute numbers [16].

However, the designing of PMs may not comply with all of these recommendations and, even if they are all fulfilled at the design stage, during its implementation or use changes in the system on which the PMs are integrated may result in PMs that do not fulfill all the above mentioned requirements [19].

III. THE PERFORMANCE MEASUREMENT PROCESS AND UNCERTAINTY SOURCES

The process usually referred in the performance measurement literature is divided into (a) design, (b) implementation, (c) use and (d) review [20], while in the field of quality management, according to Juran and Godfrey [11] those activities consist of: (i) understand the framework, (ii) plan the measurement, (iii) collect and store data, (iv) analyse, synthesise, formulate recommendations, present results and recommendations, and (v) make decision and take action.

The process of performance measurement will be detailed in this work on three stages: design (a; b; i; ii.), data collection and record (iii; c excluding analysis) and determination and analysis (iv; c only analysis).

In terms of frequency, the first stage is the less frequent and the second one the most frequent, because each new analysis requires new data.

This work will analyze the uncertainty that could be introduced in the design and implementation stage of performance measurement process and subsequently in the use and analysis stages. Finally, the overall uncertainty assessment will provide information to the performance measurement review process.

The Decision/action process is out of the scope of this work but the information about the uncertainty of a PM may be relevant to ascertain the risk associated with a given decision/action based on a PM.

Risk is a possibility and consequence of a given event. Typically, PMs are used to verify if a goal is achieved or it is used to justify actions (to put a process under control, to allocate resources, etc.). If a PM value, including measurement errors and other uncertainties, is close to a target or limit, there is a possibility that its true value could lead to a different action.

A. Performance measurement design and implementation

The process of performance measurement starts by PMs’ Design. Despite the inexistence of a universal set of rules or model to develop an ideal PM or an ideal PMS, literature suggests principles, models, frameworks and attributes or requirements of good PMs.

Designing a PM consists of defining a set of attributes, such as ([21], [22], [11] and [23]): name of PM, purpose; target; data source; PM owner; frequency of measurement; measurement method/equipment; formula; units of measurement; control/reaction limits; frequency/method of analysis; responsible of analysis; possible immediate actions; PM customer, among others.

Some of the above attributes may not be applicable. For example, if a PM’s objective is to control a process, the definition of control/reaction limits or possible immediate actions will be important attributes, but if the PM objective is to ascertain if an organisation goal, assessed by a (set of) PM is being achieved, those attribute may not be relevant.

Considering that each organisation can influence the process of designing a PM, the result of this stage could be a perfect PM design or a design with many flaws or unwanted characteristics. Operational, organizational or financial constraints may also arise during implementation and will compromise the planning and specifications defined in the design phase. Therefore several factors can, ultimately, influence (induce errors of unknown magnitude) the values of the PM. It was decided, in his work to organize them in the well-known cause-and effect or Fishbone diagram, one the basic quality tools [11]. This diagram has the ability to represent graphically, involving a team in the identification of the controllable, causes and sub-causes of a given
unwanted event, which in this case is: deficiencies at the design and implementation stage.

The controllable factors that are considered at this stage are (Fig. 1): People involved in the design and implementation stage (such as its experience and complacency); Environmental context where the measurement takes place (such as its complexity and predictability); Policy adopted by the organization (related to quality management, knowledge management and Human resources (HR) management and procedure used to design and implement the PM.

B. Performance Measurement collection and record

The Measurement activity consists of obtaining data and can be performed in different ways, such as reading a value in a measurement device that may be installed in the production process or counting the number of occurrence of a predefined event.

To use the data collected in the previous activity to calculate a PM for a given period of time, data is registered in a computer or physical datasheet. The Data Record/Transmission activity may be made in different ways: data is automatically registered when the measurement is performed (automatic record system); or the data is recorded by someone who reads the value and writes it in a computer system or datasheet (handmade record).

In both of these activities, several factors can induce uncertainty. These factors are described in detail in [24] and are arranged in Fig. 2 according to four main groups: Equipment used to measure and to record data (accuracy, measurement precision, etc.); Workplace Environment (such as luminosity, tidiness and workplace organization); Operator (such as physical and mental fitness, and complacency) and, Method (existence of instructions for measuring and recording data and its clarity).

C. Performance Measurement Determination and Analysis

The PM Determination activity consists of selecting recorded data for a specific period of time and applying a predefined expression for calculating the PM. This task may also be made automatically by a computer application or can be made manually.

Analysis consists of preparing data to make it useful to support decisions. It includes, managing missing or suspect PM values, trend analysis, summarizing data and compare with predefined values.

Similarly to the previous stages, PM determination and analysis may introduce errors in the PM value. The following factors are organized in four main groups (Fig. 3): Tool (clarity, flexibility and modus operandi of the tool used to determine and analyze the PM values), Context information (availability, clarity and accuracy of the context information used to interpret the PM values, People (such as experience and, knowledge acquiring and processing) and Method (flexibility, adaptability, availability and clarity of both determination and analysis methods).

IV. METHODOLOGY TO ESTIMATE PM UNCERTAINTY BASED ON FUZZY LOGIC

A. Input and output variables

Deficiencies generated in design, collection and record, and determination and analysis stages, introduce uncertainty in the measurement process that is reflected in greater or lesser confidence in the PM value. Once the sources of uncertainty present in each of these stages of performance measurement are identified, the uncertainty assessment team evaluates the level of these deficiencies in each of these stages and their implications on the uncertainty of the PM.

Consider that the deficiencies in the design, collection and records, and analysis stages are represented by the variables $x_d$, $x_u$ and $x_a$, respectively, and that the uncertainty in the PM is represented by the variable $y$. In mathematical terms, one could express the relationship between input variables $(x_d, x_u, x_a)$ and output variable $y$ by $y = f(x_d, x_u, x_a)$.

In many performance measurement processes there are frequently several factors that must be taken into account if we want to evaluate the deficiencies in each one of the three considered stages, and usually the implications of each one of these deficiencies are not well known.

In addition, there is not an objective scale for measuring the resulting deficiencies, represented in this work by the input variables $(x_d, x_u, x_a)$, nor for the output variable $y$.

Therefore, the most natural way to assess these variables by the team responsible for the PM uncertainty assessment
is to use linguistic terms (words) of the natural language such as “very low”, “often”, “moderate”; etc.

Concerning the function \( f \) that relates the input variables to the output variable, there is no known analytical relationship. However, the knowledge that the project team has about the implications of the deficiencies (identified at the stages level) causes on the PM uncertainty allows establishing a set of cause-consequence combinations of different levels of input variables to the output variable. In this context the general framework of fuzzy reasoning facilitates the handling of the PM uncertainty. Fuzzy logic and fuzzy systems are suitable for representing and employing knowledge that is imprecise, uncertain, or unreliable.

### B. Fuzzy Logic Method

The general Fuzzy Logic method consists of four basic components, namely:

- Fuzzification (definition of the fuzzy set of the input and output variables);
- Fuzzy rule base (definition of the rules that correlate the input variables to the output ones);
- Fuzzy inference engine (aggregation of the contributions of the rules);
- Defuzzification of the results.

#### 1) Fuzzification

It is the process of decomposing a system input variables into one or more fuzzy sets, thus producing a number of fuzzy perceptions of the input, and carrying out a mapping from real-domain variables.

In this work a fuzzy system with three input variables (intended to assess the deficiencies levels associated with variables) is designed. Each of the input variables is associated with five linguistic terms:

\( \{ \text{Very Low, Low, Moderate, High, Very High} \} \)

For the output variable more discrimination was obtained by adding another four linguistic terms obtaining thereby nine levels:

\( \{ \text{Exceptionally low, Extremely low, Very low, Low, Moderate, High, Very high, Extremely high, Exceptionally high} \} \)

Triangular membership functions are used to define the fuzzy set for each linguistic term. The membership functions are defined for the three inputs variables and the output variable (see Table 1). Graphical representation of \( x_d \) is provided in Figure 4. Similar graphs can be drawn for \( x_u \) and \( x_a \). Figure 5 represents the output variable. As shown in these figures, the input variables have their universes of discourse defined between 0 and 10, and the universe of discourse of the output variable is defined between 0 and 100.

After determining the fuzzy membership functions of all variables, the knowledge about the relationship between input and output variables is mapped through fuzzy rules.

#### 2) Fuzzy rule base

Fuzzy rules consist of a set of decisional type IF–THEN, this meaning that, the consequences occur only if the premises are real. The fuzzy rules represent the logical correlations between input and output variables and are of the following form:

\[
R^{(k)} : \text{IF } x_1 = \tilde{A}_1^k \text{ AND } ... x_n = \tilde{A}_n^k, \text{ THEN } y = \tilde{B}^k
\]

Where \( \tilde{A}_i^k (i = 1, 2, ... , n) \) and \( \tilde{B}^k \) are fuzzy sets, \( x = (x_1, x_2, x_3)^T \in U \) and \( y \in V \) are input and output linguistic variables, respectively, and \( k \) represents the number of the rules \( (k = 1, 2, ..., S) \).

The number of rules depends on the number of inputs and outputs and the desired behavior of the system. Once the rules have been established, such a system can be viewed as a non-linear mapping from inputs to outputs.

The rules are deduced from the knowledge and experience of the project team. In this case, from eq. (1) we have:

\[
\tilde{A}_i^k = (\text{EL, VL, L, M, H, VH, EH})
\]

\[
\tilde{B}^k = (\text{ExL, EL, VL, L, M, H, VH, EH, ExH})
\]

\[
R^{(1)} : \text{IF } x_d \text{ is EL and } x_u \text{ is EL and } x_a \text{ is EL, THEN } y \text{ is ExL}
\]

As in this study the system has three inputs with five linguistic terms, there are 125 (5^3) possible fuzzy rules that can be defined.

Figure 6 presents the output as a function of two input variables considering the other variable equal to its average value. Similar graphs can be obtained using other combinations of input variables.
Defuzzification is the process that transforms the output fuzzy set to crisp output by applying specific defuzzification method. There are some methods of defuzzification, but the most common is the centroid method, this calculates the center of area of the fuzzy set and uses the value at which this occurs as the defuzzified output. In this work the centroid method was used to obtain a single crisp (real) value for the output variable.

V. APPLICATION EXAMPLE

The application example concerns the manufacturing of Printed Circuit Boards (PCBs) to be used in car multimedia systems by a multinational company. In the production lines of PCBs, an automated optical inspection (AOI) system is used to control and assess the quality of the reflow soldering process. PCBs are autonomously scanned by a camera to identify a variety of soldering defects such as open circuits or short circuits. These defects are measured by the volume of solder paste placed on a given PCB position and compared with pre-defined specifications.

To reduce defects detected at the final quality control test, aligned with the company continuous improvement culture, it was decided to use a PM to be calculated at the end of the reflow soldering process: number of soldering defects per million opportunities (DPMO).

A quality team was commissioned to define the PM and the methods of collection, record, determination and analysis. The same team implemented the PM. Since the equipment, AOI, is not able to measure all the positions of soldering deposition during the cycle time, only usual critical positions are analyzed. The PCB fixation mechanism and its position when optical inspection is made is a critical factor to the quality of the measurements.

In each PCB, AOI signals and registers the number of soldering defects in the critical inspected positions in a database. The number of inspected positions is also recorded for each PCB in the same database. Daily DPMO is calculated for each shift of the company production lines. The DPMO values are controlled daily by the line manager. Weekly, in the quality team meeting, the obtained values in each line and shift are compared to each other and with the established target, and possible tendencies are checked.

In one of these weekly meetings, the analysis of the PM uncertainty was undertaken following the methodology proposed in this paper and, starting with the three cause and effects diagrams proposed, the three input variables values were defined based on linguistic terms by consensus: $x_1 =$ “low”; $x_2 =$ “medium”; $x_3 =$ “low”. The proposed rules to express uncertainty in the PM based on these input variables was presented and agreed.

Figure 7 presents the results for the DPMO studied resulting in an uncertainty level of 12.5 (out of 100). In linguistic terms it belongs to the EL and VL membership functions. This could provide a basis to compare this PM with other PMs used by the organization and to support the reviewing of existing PMs to reduce overall PM uncertainty or to improve PM quality.

VI. CONCLUSION

In this work, the process of performance measurement was defined and analyzed to identify the sources of uncertainty that may induce uncertainty in the PM. Cause-and-effect diagrams were used to graphically represent the causes and then fuzzy logic was used to quantify the uncertainty of the PM.

The causes can be assessed for any PM and some causes may be common for a given organization. In each particular case the presence and level of each source must be questioned to define its influence in the uncertainty of the considered PM.

The proposed methodology to characterize uncertainty is based on the fuzzy set theory. Three input variables are defined related to the three activities: (1) design and implementation; (2) data collection and record; and (3) analysis. These input variables result in an output variable...
that represents the level of uncertainty of a given PM. Users may edit rules to change results. The methodology is proposed and an application example is provided.

This work is part of a project that aims to develop a framework to reduce the uncertainty of performance measurement systems. Results provide a breakthrough on the process of revising and improving the Performance Measurement System allowing to identify the uncertainty sources of a PM and to quantify their contributions to uncertainty.

The proposed method allows identifying the changes that can be introduced in the performance measurement process to obtain more trustable values for the key PMs used in decision-making. The knowledge of uncertainty associated with a PM also allows considering, in decision-making process, the risk due to lack of data quality.

REFERENCES