Self-sensing concrete with nanomaterials

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Abstract: Conductive concrete containing nano carbon black (NCB) and carbon fibre (CF) to enable the self-diagnosis of strain and damage was studied. The effect of NCB and CF on workability, mechanical properties and fractional change in resistance (FCR) in fresh and hardened concrete was analysed. The relationship between the FCR, the strain of initial geometrical neutral axis (IGNA) and the degree of beam damage was established. The results showed that the relationship between the FCR and the IGNA strain can be described by the First Order Exponential Decay function, and that the slope of this function reflects the sensitivity of conductive concrete. Based on the above relationship and damage mechanics theory, internal damage to the concrete is indicated by the relationship between the degree of damage and resistance. This self-sensing of strain in conductive concrete can be applied in monitoring damage to flexible components.

Key words: conductive concrete, nano carbon black, carbon fibre, self-diagnosing of damage, fractional change in resistance, strain.

4.1 Introduction

Degradation, cumulative structural damage or material resistance are common reasons for the failure of conventional concrete structures such as dams and bridges which undergo differing levels of load, fatigue or corrosion. In order to prevent the possibility of sudden failure and to prolong the service life of concrete structures, the study of cumulative damage has concentrated on strain behaviour and fatigue process (Li and Ou, 2007; Ou, 1996). Monitoring is valuable for structural safety and the application of conductive cementitious composite materials was reported by Wen and Chung (2000, 2004, 2005, 2006, 2007), using electric resistance measurement to monitor strain and damage.

Over the last decade, nanomaterials have been used as smart fillers for a broad range of multifunctional composites as well as in strain or damage sensors (Li *et al.*, 2008; Chung, 2012). This chapter considers the addition of both nano carbon black (NCB) and carbon fibre (CF) as conductive phases which enhance electrical conductivity and produce diphasic electric conduc-

tive concrete. This enables resistivity measurements to be carried out and may be used to analyse variations of strain or stress in structural components, thus making possible the early evaluation of damage without the need to embed sensors. Conduction concrete also has a wide application in the electromagnetic shielding of vital equipment and de-icing of airfields and highways (Yehia and Tuan, 1999).

The addition of short CF to concrete creates continuous conductive pathways which transmit current, playing a fundamental role in the electrical transport process. The enhanced electrical conductivity of concrete with CF also decreases shrinkage and cracking, so improving durability and resistance to freezing. In addition, it does not require a large quantity of water. The addition of NCB reduces cost by improving electrical conductivity and the toughness of the aggregate interface within the concrete matrix. It also provides a filler effect which enhances the density of the matrix (Cai and Chung, 2007). Due to the extremely small size of NCB compared to traditional carbon fibre, it penetrates the matrix in carbon fibre reinforced composites. This connects the conductive pathways to form conductive networks which further improve electrical conductivity (Li *et al.*, 2006, 2008). Initial and evolving strain in concrete may cause damage which breaks conductive pathways or networks, resulting in a change of electric resistance. The combined use of NCB and short CF provides conductive concrete with effective mechanical properties.

Damage to cement-based material may change its electric resistance, as manifested in elastic tension, plastic deformation and cracking. Concrete components with coarse aggregate are often subjected to differences in loading (e.g., compression, tension and bending), and may also experience various stages of load-deformation, including pre- and post-cracking behaviour. Studies on strain sensing in carbon fibre reinforced geopolymer concrete under conditions of bending and compression have been reported. However, studies on concrete beams with diphasic electrical conduction admixtures for diagnosis of damage caused by bending are still very rare. There are several problems in the study of conductive concrete beams. The electric characteristics must be suitable for a particular application without degradation of the workability of fresh concrete or detriment to the mechanical behaviours of hardened concrete.

Based on investigations into the effect of NCB and CF on the workability, compression strength and flexural strength of concrete, a large number of concrete beams reinforced with conductive material were investigated experimentally to study the damage and FCR under varied loading levels. The purpose of this work was to analyse the effect of NCB or CF, and especially the hybrid use of NCB and short CF, as diphasic conductive materials on the FCR of concrete beams. It was also concerned with the relationship between FCR, strain and degree of damage in concrete beams

undergoing bending in the pre-cracking region. The relationship between FCR and the strain of initial geometrical neutral axis (IGNA) was established by regression analysis. Based on damage mechanics theory and the relationship mentioned above, the correlation between the degree of damage and the FCR was established. The results show that the relationship between FCR and the strain of the IGNA in concrete beams can be effectively described by the First Order Exponential Decay curve before cracking.

4.2 Studying conductive admixtures in concrete

4.2.1 Materials

The nano carbon black (Fig. 4.1) used in the experiments was a super-conductive form of porous agglomerates of carbon particles with an average size of 60 nm and a density of 0.3–0.5 g/cm³. The carbon fibre used (Fig. 4.2) in the conductive phase was asphalt base short carbon fibre with a diameter of 12–15 μm and a length of 6 mm. Its density was 1.55–1.60 g/cm³. The properties of NCB and CF used are shown in Tables 4.1 and 4.2.

In order to analyse the effect of conductive admixtures on the concrete, plain concrete samples without CF and NCB were prepared as references. The design mixture of the plain concrete is shown in Table 4.3. The type of cement used was CEM I 42.5, the W/B was 0.45, and a water reducing agent (WR) was used in the amount of 1.0% by mass of binder. The 28d compressive strength was 43.6 MPa. The NCB content with a particle size of *ca.* 30–90 nm (Fig. 4.1) was between 0.1% and 0.4% by mass of binder (0.53–

4.1 Nano carbon black and particle size using high resolution field emission SEM.

4.2 Carbon fibre.

Table 4.1 Properties of NCB

Particle size (nm)	Density (g/cm ³)	Volume resistance (Ω .cm)	Surface resistance (Ω)
30–90	0.3–0.5	2.3	30.6

Table 4.2 Properties of CF

Density (g/cm ³)	Diameter (10 ⁻⁶ mm)	Length (mm)	Tensile strength (N/mm ²)	Resistivity (m Ω .cm)
1.55–1.6	12–15	6	3000	3–7

Table 4.3 Design mixture of reference concrete

Cement (kg/m ³)	Fly ash (kg/m ³)	Fine aggregate 1–4 mm (kg/m ³)	Coarse aggregate 5–10 mm (kg/m ³)	Water (kg/m ³)	SP (kg/m ³)
370	160	733	733	240	5.33

Table 4.4 Comparison of the dosages of the conductive admixtures

Serial number		NCB (mass/binder %)	CF (mass/binder %)
Plain concrete (concrete without conductive admixture)	PC	0	0
Concrete containing NCB	NCB 01	0.1%	0
	NCB 02	0.2%	0
	NCB 03	0.3%	0
	NCB 04	0.4%	0
Concrete containing CF	CF 04	0	0.4%
	CF 08	0	0.8%
	CF 10	0	1.0%
	CF 13	0	1.3%
	CF 16	0	1.6%
Concrete containing BF (NCB and CF)	BF 14	0.1%	0.4%
	BF 18	0.1%	0.8%
	BF 24	0.2%	0.4%
	BF 28	0.2%	0.8%

2.12 kg/m³). The carbon fibre content with a diameter of 12–15 μm and a length of 6 mm (Fig. 4.2) was between 0.4% and 1.6% by mass of binder (2.12–8.48 kg/m³). Where CF was used (both alone and in combination with NCB), methyl-cellulose was used as a dispersing agent along with a defoamer to optimize the dispersion of carbon fibre in the concrete (Wen and Chung, 2007). Methyl-cellulose and defoamer were not used in the absence of carbon fibre. The defoamer dosage was 2.13 kg/m³, and the methyl-cellulose content was between 2.13 and 8.52 kg/m³. The series of electric conductive concrete samples (CF only, NCB only, hybrid use of CF and NCB) and different contents of conductive phases are listed in Table 4.4.

4.2.2 Specimen preparation and testing set-up

Mixing was carried out using a mechanical concrete mixer. Methyl-cellulose was dissolved in water and the defoamer and CF (if applicable) were added and stirred manually for around two minutes. The methyl-cellulose mixture, fine aggregate, coarse aggregate, cement, fly ash, water, NCB and superplasticizer were then mixed for five minutes. The mixture was poured into oiled moulds and an external electric vibrator was used to facilitate compaction and to decrease the quantity of air bubbles. The specimens were removed from the moulds after a day and four electrical contacts in the form of conductive adhesive tape were wrapped around the specimens. Based on

the four-pole method of electric resistance measurement, contacts A and D were used for passing the current while contacts B and C measured the voltage (Cai and Chung, 2006). Finally, carefully prepared specimens were cured at room temperature for 28 days. All the beams prepared for testing measured $100\text{ mm} \times 100\text{ mm} \times 400\text{ mm}$. The dimensions and electrical contacts details of all beams are shown in Fig. 4.3. The result represents the average across three beams.

4.2.3 Test methods

The four-pole method was adopted for measuring resistance (Tian and Hu, 2012). A hydraulic servo testing machine (MTS Model 810) was used. The close-loop test was controlled by displacement and the deformation rate at the mid-span was $0.2 \pm 0.02\text{ mm}$ per minute up to the specified end-point deflection which was 3 kN larger than that of the previous loading level. The two possibilities in loading and load-time relationships are illustrated in Fig. 4.4.

4.3 Specimen configuration for measuring resistance.

4.4 Loading histories of the beam: (a) load-time relationship I, (b) load-time relationship II.

4.5 Arrangement of measuring points.

Six strain gauges were applied for measuring the longitudinal strain, two of which (Strain 2(5)) were used on each side of the two opposing surfaces to measure the strain of initial geometrical neutral axis (IGNA) under the externally applied load N .

During the loading process, the strain near the top of the concrete beam in the compression zone, the strain of IGNA and the tensile strain near the bottom of the beam were measured by strain gauges. The resistance of concrete beam was measured simultaneously. The IGNA strains were then obtained by strain gauges (2) and (5). The resistance of the beams was continuously measured during loading by using the four-pole method described above. Other experimental instruments included an AC stabilized voltage supply, IMC Intelligence Data Collecting System, a fixed resistor and an AC/DC converter. A schematic view of the beam under loading with current and voltage electrodes is illustrated in Fig. 4.5. Rubber joints were placed under the support points during the experiment (see Fig. 4.5) in order to isolate the concrete beam from the loading frame.

4.3 Influence of conductive admixtures on the mechanical properties of concrete

4.3.1 Influence on workability and compression strength

The workability of high flowable fresh concrete, with and without conductive admixture, has been evaluated by measuring the slump flow. The experimental results of workability are listed in Table 4.5. The factor d represents

Table 4.5 Content of conductive admixture, slump flow, compressive strength and flexural strength

Mixture/samples		Content of NCB (kg/m ³)	Content of CF (kg/m ³)	Slump flow, <i>d</i> (mm)	Compressive strength (<i>f_{cu}</i>) (MPa)	Flexural strength (<i>σ_u</i>) (MPa)
Plain concrete	PC	0	0	620	42.55	4.44
Concrete with NCB only	NCB01	0.3733	0	600	43.53	4.52
	NCB02	0.7467	0	540	43.67	4.71
	NCB03	1.1200	0	390	44.17	4.79
	NCB04	1.4933	0	330	45.45	4.83
Concrete with CF only	CF04	0	1.4933	590	43.49	5.10
	CF08	0	2.9866	570	44.37	5.19
	CF10	0	3.7333	550	45.10	5.22
	CF13	0	4.8533	520	44.6	5.30
	CF16	0	5.9733	500	44.76	5.31
Concrete with BF (NCB + CF)	BF14	0.3733	1.4933	590	43.78	4.90
	BF18	0.3733	2.9866	540	44.01	4.94
	BF24	0.7467	1.4933	480	43.54	5.02
	BF28	0.7467	2.9866	410	43.98	5.13

the average diameter in the slump flow test. It may be seen from Table 4.5 that fresh PC (plain concrete without any conductive admixtures) corresponds well to the requirements of self-compacting concrete. There is very good flowability and no segregation. However, the workability of fresh concrete declines with an increase in NCB or CF content. The slump flow of BF28 is only about 410 mm. This means that the content of diphasic conductive admixtures (0.747 kg/m³ (NCB) + 2.99 kg/m³ (CF)) is less than the lower limit (450 mm) of the workability of highly flowable concrete. The flow behaviour of NCB03, NCB04, and BF28 is much less fluid than that of other mixtures due to the relatively high content of CF and NCB.

The average values of compressive strength *f_{cu}* and flexural strength *σ_u* after 28 days may be found in Table 4.5. The increment of compression strength ranges between 2.2% and 6.2%. This indicates that the addition of NCB, CF and BF shows some positive effect on the compressive strength of concrete, but does not amount to a significant trend of improvement.

4.3.2 Influence of conductive admixtures on flexural strength

When a beam is subjected to bending, strains are produced. These strains create compression stress at the top of the beam and tension at the bottom. The load is applied until a crack is imminent. If the beam section does not

crack, then the ordinary elastic beam theory applies. The effects of conductive admixtures on flexural strength $\sigma_u = M_u/W$ (where M_u is ultimate bending moment and W is the section modulus) are illustrated in Table 4.5. It may be seen that the flexural strength of a concrete beam increases with the addition of conductive admixtures. Amongst these, the samples containing only NCB show a smaller increment of flexural strength as the NCB dosages are increased (between 1.8% and 8.8%). However, there is a clear increase in flexural strength when both the CF and BF dosages are increased. When compared with a PC beam without any conductive admixture, the increment of flexural strength in beams containing CF and BF is between 10% and 19%.

4.4 Influence of conductive admixtures on the electrical properties of concrete beams

As described above, flexural strains are produced when a beam is subjected to bending. During the loading process, the strain near the top of the beam in the compression zone, the strain of initial geometrical neutral axis (IGNA) and the tensile strain near the bottom of the concrete beam (Fig. 4.5) are measured by strain gauges. The resistance of the beam is measured simultaneously. The strains of IGNA are obtained by strain gauges (2) and (5), and the FCR at each loading stage can also be measured. The relationship between the force and electrical fields is used to investigate stress and strain behaviour through analysis of physical quantities such as current, voltage and resistance (Wu, 2005).

4.4.1 Feasibility of relationship between force field and electric field

It is difficult to measure quantities such as stress or current density directly. However, there are some experimentally measurable physical quantities, such as resistance in the electric field and strain in the force field. The stress can be calculated by stress-strain curves, while current and current density can be calculated by Ohm's law. Once the resistance-strain correlations are established through experimentation, the relationship between the force and electric fields is determined, so making it possible to obtain the immeasurable quantities in the force field with the measurable voltage and resistance of the electric field. The corresponding parameters of the electric field and the force field are illustrated in Table 4.6.

The Laplace Equation [4.1] is satisfied for both the electric and force fields:

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \quad [4.1]$$

Table 4.6 Comparison of electrical and mechanical parameters

Electric field variable	Force field variable
Current: I	Load: N
Voltage: V	Deformation: U
Current density: $i = I/A = Ce$	Stress: $\sigma = N/A = E\epsilon$
Generated voltage: $e = V/L$	Strain: $\epsilon = U/L$
Ohm's law: $I = V/R$	Hooke's law: $N = U/\delta = EA\epsilon$
Resistivity: $\rho = RA/L$	Stiffness coefficient: $B = EA/L = 1/\delta$
Conductivity: $C = 1/\rho = L/RA$	Elastic modulus: $E = L/(A\delta)$
Sensitivity: $\lambda = \Delta R/(R\epsilon)$	Damage degree: $D = \Delta E/E$

For the electric field:

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0 \quad [4.2]$$

For the force field:

$$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} (\sigma_x + \sigma_y + \sigma_z) = 0 \quad [4.3]$$

where x , y and z are space coordinates; ϕ and σ are respectively the potential function and stress. The Laplace equations presented here are based on the assumption that both the electric field (see Fig. 4.6a) and the force field (see Fig. 4.6b) comply with the same boundary conditions. In the force field, the beam is subjected to three loading points (A and E are the support points, and C is the mid-span point with an applied external load N , see Fig. 4.5 and Fig. 4.6b). In the electric field, the beam is taken as being in a similar situation as voltage is provided through three points, B , C and D . As illustrated in Fig. 4.6a, electric potential points B and D are used to provide the same voltage, while point C at the mid-span is the ground terminal. Thus if the beam section does not crack (the ordinary elastic beam theory applies), the boundary condition of the electric field will be comparable with that of the force field.

The variation of strain on the IGNA, along with the load, is shown in Fig. 4.7 as a function of time during the loading process prior to cracking. Figure 4.8 shows the relationship between the resistance and strain of IGNA obtained simultaneously from the same specimen. It can be seen that the resistance decreases along with the strain of the IGNA due to the sensitivity of conductive concrete as previously reported. The relationship between the IGNA strain in the force field and the resistance in the electric field can therefore be established.

4.6 Comparison of loading boundary conditions between electric and force fields: (a) model of the electric field, (b) model of the force field.

4.4.2 Relationship between strain and FCR (Self-diagnosing of strain)

Figures 4.9–4.12 illustrate the relationship between FCR and the strain of IGNA (ϵ_2) in concrete beams. The effects of various conductive admixtures on these relationships can be also observed. It may be seen that the relationship between the FCR and the strain of IGNA corresponds well with the First Order Exponential Decay function, which may be expressed by:

$$Y = m \exp(-X/n + p) \quad [4.4]$$

where m , n and p are constant parameters corresponding to the type and amount of the electric conductive phase and the variable X is the strain of

4.7 Variation of strain of IGNA (■) with load (•) vs time before cracking.

4.8 Resistance (•) vs strain of IGNA (■) and time.

IGNA; the unit of X is in $\mu\epsilon$, and the FCR is the percentage of Y . The applied parameters and the correlation coefficient C_R^2 are illustrated in Table 4.7. The correlation coefficients of all beams in Table 4.7 range from 0.5 to 0.978. From Figs 4.9–4.12 and Table 4.7, it may be seen that:

4.9 Relationship between FCR and ϵ_2 of PC.

4.10 Relationship between FCR and ϵ_2 of NCB 03.

- The correlation coefficient C_R^2 of a plain concrete (PC) beam is only 0.50. This means that in a PC beam without conductive admixtures, the tested value is not strong in relation to the prediction in Eq. [4.4].
- The correlation coefficient C_R^2 of other beams with conductive materials such as NCB, CF or diphasic electric conductive materials (NCB + CF) is higher than 0.76. Hence, the relationship between FCR and strain of IGNA is quite strong when correlated with Eq. [4.4].

4.11 Relationship between FCR and ϵ_2 of CF 13.

4.12 Relationship between FCR and ϵ_2 of BF 28.

- The correlation coefficient C_R^2 of NCB03, NCB04, CF10, CF13, BF14 and B24 is higher than 0.9. The relationship between FCR and the strain of IGNA is therefore very strong when correlated with Eq. [4.4], and the self-diagnosis of damage could be more effective, particularly in concrete components with the conductive admixtures suggested above.
- The curves in Figs 4.9–4.12 demonstrate a monotonically decreasing relationship between FCR and the strain of IGNA.

Table 4.7 Fitted parameters of regression equation

Serial number	Constant <i>m</i>	Constant <i>n</i>	Constant <i>p</i>	Correlation coefficient C_R^2
PC (NCB0% + CF0%)	16.21	33.06	-10.33	0.50033
NCB01 (NCB0.1% + CF0%)	32.31	47.48	-21.05	0.82466
NCB02 (NCB0.2% + CF0%)	34.57	5.21	-39.11	0.76742
NCB03 (NCB0.3% + CF0%)	56.89	85.79	-44.85	0.92748
NCB04 (NCB0.4% + CF0%)	23.04	17.57	-25.56	0.90907
CF04 (NCB0% + CF0.4%)	27.87	13.00	-29.74	0.79566
CF08 (NCB0% + CF0.8%)	20.83	19.26	-21.25	0.83359
CF10 (NCB0% + CF1.0%)	25.55	12.39	-27.41	0.97752
CF13 (NCB0% + CF1.3%)	12.39	14.84	-11.00	0.9471
CF16 (NCB0% + CF1.6%)	42.96	15.75	-52.68	0.82598
BF14 (NCB0.1% + CF0.4%)	14.88	95.04	-12.56	0.95126
BF18 (NCB0.1% + CF0.8%)	106.63	83.70	-82.74	0.77459
BF24 (NCB0.2% + CF0.4%)	12.98	34.94	-15.24	0.90456
BF28 (NCB0.2% + CF0.8%)	13.45	68.90	-16.11	0.8813

4.4.3 Sensitivity of conductive concrete

The ability of a structural material to sense its own strain (i.e., sensitivity) is an attractive attribute of smart structures. The sensitivity of conductive concrete may be characterized by the gauge factor (λ) which is defined as the fractional change in resistance per unit strain (Chung, 2012; Tian and Hu, 2012). Hence λ is equal to the slope of Eq. [4.1] and may be expressed by:

$$\lambda = |0.01Y'| = \left| \frac{m}{100n} \exp(-X/n) \right| \quad [4.5]$$

where m and n are both constant parameters and the variable X is the strain of the initial geometrical neutral axis (IGNA) as in Eq. [4.4].

It may be seen from Eq. [4.5] that the value of λ decreases in terms of exponential decay function as the strain IGNA increases, i.e., the development of strain leads to a degradation of sensitivity in electric conductive concrete.

4.5 Strain and damage in concrete beams (self-diagnosing of damage)

The damage done to a beam prior to the concrete cracking is discussed in this section and is based on the theory of damage mechanics (Cai and Cai, 1999). The problem may be simplified as uni-dimensional damage and only

the damage in the tension area is analysed. The effective tensile stress $\tilde{\sigma}_t$ in the tension area is expressed by:

$$\tilde{\sigma}_t = \frac{\sigma_t}{1-D} = E\varepsilon_t, (\sigma_t \geq 0, D \geq 0) \quad [4.6]$$

where D and E are respectively the degree of damage and the elastic modulus; ε_t and σ_t are respectively the strain and stress of the extreme tension fibre at the bottom of the concrete beam.

The effective compressive stress $\tilde{\sigma}_c$ in the compressive area is given by:

$$\tilde{\sigma}_c = \sigma_c = E\varepsilon_c, (\sigma_c \leq 0, D = 0) \quad [4.7]$$

where ε_c and σ_c are respectively the strain and stress of the extreme compression fibre at the top of the concrete beam.

When the effective stress $\tilde{\sigma} > 0$ (tension stress), we have $D = \frac{\tilde{\sigma}}{k}$, where k is the damage modulus. When $\tilde{\sigma} \leq 0$, we have $D = 0$. Based on the strain equivalence hypothesis, the following is obtained:

$$\sigma = (1-D)\tilde{\sigma} = (1-D)E\varepsilon = \left(1 - \frac{\tilde{\sigma}}{k}\right)E\varepsilon = \left(1 - \frac{E\varepsilon}{k}\right)E\varepsilon \quad [4.8]$$

where ε and σ are respectively the strain and stress of the concrete matrix.

If the beam is in the elastic stage, the degree of damage in the concrete $D = 0$, then $\sigma = E\varepsilon$. After the elastic stage, the stress state of the tension zone tends to be elasto-plastic due to the redistribution of stress before cracking. The correlation between ε and σ may be derived from Eq. [4.8] and expressed by:

$$\frac{d\sigma}{d\varepsilon} = E - \frac{2E^2\varepsilon}{k} \quad [4.9]$$

At the peak value of stress, the first derivative of stress should be equal to zero as described in Eq. [4.10]:

$$\frac{d\sigma}{d\varepsilon} = 0 \quad [4.10]$$

In this event, the critical cracking stress is σ_{cr} and the degree of damage D is equal to D_{cr} . From Eq. [4.9] and [4.10], we get $E\varepsilon = \frac{k}{2}$. Replacing $E\varepsilon$ in Eq. [4.8] gives the critical cracking stress $\sigma_{cr} = \frac{k}{4}$.

When $D = \frac{\tilde{\sigma}}{k}$, σ_{cr} can be evaluated as $\sigma_{cr} = \frac{k}{4} = (1-D_{cr})D_{cr}k$, from which $D_{cr} = 0.5$ is obtained. Therefore, the effective critical cracking stress $\tilde{\sigma}_{cr}$ may be expressed as:

$$\tilde{\sigma}_{cr} = D_{cr} k = \frac{k}{2} \quad [4.11]$$

The IGNA of the beam section may move up to the compression area as the tension area of the concrete beam is damaged and behaves in an approximately plastic manner. The depth of the beam section ($2h$) and the displacement of IGNA (the distance between the actual geometrical neutral axis and the initial geometrical neutral axis y_0) are illustrated in Fig. 4.13. The width of the beam section is b . During the loading process, the stress pattern in the compression zone changes continuously. Based on the plane section assumption, the relationship of stresses and strains may be described as follows:

$$\tilde{\sigma}_t = \frac{y - y_0}{h - y_0} \tilde{\sigma}_3 \quad [4.12a]$$

$$\tilde{\sigma}_1 = \frac{h + y_0}{y_0 - h} \tilde{\sigma}_3 \quad [4.12b]$$

$$\varepsilon_3 = \left(1 - \frac{h}{y_0}\right) \varepsilon_2 \quad [4.12c]$$

where ε_1 , ε_2 and ε_3 are respectively the concrete strain at the top of the beam (outer fibre of the compression zone), the strain of the IGNA and the strain of the extreme tension fibre at the bottom, and $\tilde{\sigma}_1$, $\tilde{\sigma}_3$ are the corresponding effective stresses at the top and bottom of the beam section, respectively.

The resultant compression force N_c and the resultant tension force N_t in the cross section of the beam may be expressed as follows:

4.13 Flexural damage of rectangular concrete beam.

$$N_c = \frac{1}{2} b(h + y_0) \tilde{\sigma}_1 = \frac{b(h + y_0)^2}{2(y_0 - h)} \tilde{\sigma}_3 \quad [4.13a]$$

$$N_t = \int_{y_0}^h dN_t = \frac{1}{6} b(h - y_0) \left(3 - \frac{2\tilde{\sigma}_3}{k} \right) \tilde{\sigma}_3 \quad [4.13b]$$

The internal moment M_c caused by N_c and the internal moment M_t caused by N_t may be derived and expressed as follows:

$$M_c = -\frac{1}{6} b(h + y_0)(2h - y_0) \tilde{\sigma}_1 = \frac{b(h + y_0)^2 (2h - y_0)}{6(h - y_0)} \tilde{\sigma}_3 \quad [4.14a]$$

$$M_t = \int_{y_0}^h y dN_t = \frac{1}{12} b(h - y_0) \left[4h + 2y_0 - \frac{\tilde{\sigma}_3}{k} (3h + y_0) \right] \tilde{\sigma}_3 \quad [4.14b]$$

Based on the equilibrium condition of the section, the following equations may be established:

$$N_c = N_t \quad [4.15a]$$

$$M = M_c + M_t = \frac{NL}{4} \quad [4.15b]$$

where M is the external bending moment.

From Eqs [4.12]–[4.15], the effective stress $\tilde{\sigma}_1$, $\tilde{\sigma}_3$ and degree of damage D of the extreme tension fibre at the bottom of the beam may be derived and expressed as follows:

$$\tilde{\sigma}_1 = \frac{6hy_0(h + y_0)}{(h - y_0)^3} k \quad [4.16]$$

$$\tilde{\sigma}_3 = \frac{-6hy_0}{(h - y_0)^2} k \quad [4.17]$$

$$D = \frac{\tilde{\sigma}_3}{k} = \frac{-6hy_0}{(h - y_0)^2} \quad [4.18]$$

As cracks form in the tensile zone of a concrete beam, failure occurs and the load bearing capacity of the beam will depend on the effective tensile stress $\tilde{\sigma}_3$. When the effective tension stress of concrete $\tilde{\sigma}_3$ reaches the critical cracking stress $\tilde{\sigma}_{cr} \left(\tilde{\sigma}_3 = \tilde{\sigma}_{cr} = \frac{k}{2} \right)$, then a beam without reinforcement will fail. From Eqs [4.13] and [4.14], the displacement of IGNA $(y_0)_{cr}$ and N_{cr} may also be described as follows:

$$(y_0)_{cr} = -0.102h \quad [4.19]$$

$$N_{cr} = \frac{0.944kbh^2}{L} \quad [4.20]$$

In the equations above, the factors such as strain (ϵ_1 , ϵ_3) and the corresponding effective stress ($\tilde{\sigma}_1$ and $\tilde{\sigma}_3$) may be given as follows:

$$\epsilon_1 = \frac{6N_{cr}Ly_0(h+y_0)}{0.944Ebh(h-y_0)^3} \quad [4.21]$$

$$\tilde{\sigma}_1 = \frac{6N_{cr}Ly_0(h+y_0)}{0.944bh(h-y_0)^3} \quad [4.22]$$

$$\epsilon_3 = \frac{-6N_{cr}Ly_0}{0.944Ebh(h-y_0)^2} \quad [4.23]$$

$$\tilde{\sigma}_3 = \frac{-6N_{cr}Ly_0}{0.944bh(h-y_0)^2} \quad [4.24]$$

The relationship between the strain of IGNA (ϵ_2) and the degree of damage D may be written as follows:

$$\epsilon_2 = \frac{N_{cr}L}{0.944Ebh^2} \cdot D \frac{1 - \sqrt{1 - \frac{2}{3}D}}{2} \quad [0 \leq D \leq 0.5] \quad [4.25]$$

It may be seen that the degree of damage D increases in a monotonic manner with an increase in the strain of IGNA ϵ_2 .

Where R_0 denotes the initial electric resistance of a concrete beam before loading, R denotes the electric resistance of a beam subjected to external loading at different times and the First Order Exponential Decay function $Y = m \exp(-X/n) + p$, Y is replaced by $100\Delta R/R_0$ and X by ϵ_2 , the relationship between the strain of IGNA (ϵ_2) and the FCR is obtained, which may be demonstrated as:

$$\epsilon_2 = -n \ln \left[\frac{100(R - R_0)}{mR_0} - \frac{p}{m} \right] / 10^6 \quad [R \leq R_0] \quad [4.26]$$

It may be seen that ϵ_2 increases with an increase in the absolute value of FCR.

The relationship between the degree of damage (D) and the FCR may be written as:

$$-n \ln \left[\frac{100(R - R_0)}{mR_0} - \frac{p}{m} \right] / 10^6 = \frac{N_{cr}L}{0.944Ebh^2} \cdot D \frac{1 - \sqrt{1 - \frac{2}{3}D}}{2} \quad [4.27]$$

It may be seen that the degree of damage D increases monotonically with an increase in the absolute value of FCR.

When the elastic modulus E , beam dimensions (b , $2h$, L) and the cracking load (N_{cr}) are given, the electrical resistance (both the initial R_0 before loading and R under loading) can be measured. The stress-strain state

subjected to the cracking load may therefore be calculated according to Eqs [4.21]–[4.25]. The displacement of IGNA $(y_0)_{cr}$ under the cracking load is obtained from Eq. [4.19] and the degree of damage D of the concrete beam under the cracking load can be evaluated according to Eq. [4.27].

4.6 Diphasic electrical conductive materials

Specimen BF28 is taken as an example for diphasic electric conductive materials (the combination of NCB0.2% and CF0.8%). The beam dimensions are given as follows: section width (b) = 100 mm, section depth ($2h$) = 100 mm, beam length (L) = 300 mm. The loading history of the beam with BF28 is illustrated in Fig. 4.4b. The elastic modulus (E) is 3.0×10^4 N/mm²; the cracking load (N_{cr}) is 14.5 kN; the parameters m (=13.45), n (=68.90) and p (= -16.11) may be obtained from Table 4.7 before the beam is subjected to loading N (at the beginning ($t = 0$)), $R_0 = 1190 \Omega$; and $\varepsilon_2 = 0$, $y_0 = 0$, $D = 0$. When the beam is subjected to load N , concrete damage occurs and the resistance of the electric conduction concrete t $R(t)$, can be measured at any time. The values of the degree of damage D , the stresses and strains and the displacement of IGNA y_0 of diphasic electric conductive concrete beam BF28 at different load times are calculated using Eqs [4.21]–[4.27] and summarized in Table 4.8.

From Table 4.8 and Fig. 4.4, the following points may be observed:

- Prior to cracking in a flexural beam, the electrical resistance (R) usually declines with an increase in the loading magnitude and duration.
- The absolute value of the FCR in a flexural beam increases with an increase in the loading magnitude and duration.
- Both the degree of damage (D) and the strain of IGNA (ε_2) increase with an increase of the absolute value of FCR. At a time of 1655 seconds

Table 4.8 Results of measured resistance, calculated degree of damage, and the effective stress and strain of BF28

Variable factors	Calculation results		
Time t (s)	269	475	836
R (Ω , measured)	1142	1119	1095
IFCRI (%)	4	6	8
D (%)	26.00	42.18	49.53
ε_2 (10^{-6})	7.23	19.67	34.86
y_0 (mm)	-2.38	-4.12	-4.91
$\tilde{\sigma}_1$ (N/mm ²)	-4.35	-6.58	-8.08
$\tilde{\sigma}_3$ (N/mm ²)	4.78	7.76	10.18
ε_1 (10^{-6})	-144.98	-219.22	-269.42
ε_3 (10^{-6})	159.47	258.59	339.17

after loading, $D = 0.486$, which is close to $D_{cr} = 0.5$, indicating that the cracking load N_{cr} is almost reached.

- As the strain of IGNA (ε_2) increases with the loading magnitude, the FCR increases gradually with the increase of D . This provides convincing evidence that the suggested formula $Y = \text{mexp}(-X/n) + p$ effectively meets the relationship between ε_2 and FCR.
- The absolute value of the displacement of IGNA (y_0) increases gradually with increased absolute values of FCR and D .
- Other factors such as $\bar{\sigma}_1$ and $\bar{\sigma}_3$, ε_1 and ε_3 increase with an increase in FCR and D .

4.7 Conclusions

A large number of experimental investigations have been carried out on the workability, electric properties and mechanical behaviour of concrete containing conductive materials. The effects of NCB, CF and diphasic BF on the relationship between fractional change in resistance, strain and degree of damage prior to cracking have been analysed. A relationship between the FCR and the strain of IGNA (ε_2) is suggested. The results have led to the following conclusions:

1. Concrete conductivity increases with an increase in dosages of NCB and CF. However, the workability of fresh concrete is an important precondition for selecting the types and content of the conductive materials.
2. Workability declines with an increase in NCB or CF content and the upper limit of conductive materials is determined mainly by workability, not by conductivity. This point had not been taken into consideration in previous investigations.
3. The flexural strength of a beam usually increases with an increase in CF and BF content. However, only the addition of NCB exerts any influence on the flexural strength.
4. The strain of IGNA (ε_2) is a function of FCR. Prior to the concrete cracking, the First Order Exponential Decay function agrees well with the relationship between ε_2 and $|FCR|$ during the loading process of CFRC beams.
5. Both the degree of damage (D) and the strain of IGNA (ε_2) increase with an increase in FCR.
6. All the mechanical factors in concrete containing conductive admixtures decline with an increase in FCR and D .
7. Results from examples evaluating the effect of diphasic electric conductive materials on the capacity for the self-diagnosis of strain and damage have been validated.

4.8 References

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