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NIPE WP 09/ 2014

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# Behavior-Based Price Discrimination with Retention Offers \*

Rosa-Branca Esteves

Universidade do Minho (EEG) and NIPE

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## Abstract

This paper is a first step in investigating the competitive and welfare effects of behavior-based price discrimination (BBPD) in markets where firms have information to employ retention strategies as an attempt to avoid the switching of their clientele to a competitor. We focus on retention activity in the form of a discount offered to a consumer expressing an intention to switch. When retention strategies are allowed, forward looking firms anticipate the effect of first period market share on second period profits and price more aggressively in the first-period. Thus, first period equilibrium price under BBPD with retention strategies is below its non-discrimination counterpart. This contrasts with first period price above the non-discrimination level if BBPD is used and retention activity is forbidden. Regarding second period prices, the use of retention offers increase the price offered to those consumers who do not signal an intention to switch; the reverse happens to those consumers who decide to switch after being exposed to retention offers. As in other models where consumers have stable exogenous brand preferences, the instrument of BBPD is bad for profits and welfare but good for consumers. BBPD with the additional tool of retention activity boosts consumer surplus and overall welfare but decreases industry profit.

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# 1 Introduction

In markets with repeated purchases firms frequently use the consumers' purchase history to quote different prices to their own previous customers and to those who bought from a rival before. When price discrimination is permitted and trade among consumers is not feasible, firms may want to price low to poach their rival's customers and price high to their own customers. This form of price discrimination, termed behavior-based price discrimination (henceforth BBPD), sometimes also called price discrimination based on purchase history or dynamic pricing, is widely observed in many markets. In the communications markets, for instance, firms frequently offer a lower price to a customer who has been using a competitor's service. Similar pricing strategies are employed in other markets such as supermarkets, web retailers, credit cards, banking services and electricity and gas.<sup>1</sup>

Although this type of competitive price discrimination has received much attention in the economics literature in recent years,<sup>2</sup> the literature has hitherto focused on the assumption that firms do not react to the rivals' poaching offers. Interestingly, in some of the markets where firms often discriminate between their own and the rivals' consumers, the use of retention offers as an attempt to avoid customer poaching and switching has become a widespread business practice. A recent report by the regulator and competition authority for the UK communications industries (Ofcom, 2010) refers that retention offers have been increasingly used by firms operating in markets in which the switching process is the Losing Provider Led (LPL). The LPL regime is currently in place in the UK for switching mobile telephony or broadband services and operates as follows. Consumers wishing to switch their mobile telephony services must contact their existing provider and request a Porting Authorization Code (PAC) which they must communicate to their new provider in order to complete the switch.<sup>3</sup> The same procedure also applies for switching broadband services, in which case the required code is the Migrations Authorization Code (MAC).

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<sup>1</sup>A recent report by the Office of Gas and Electricity Markets (Ofgem (2008)), the regulator for Britain's gas and electricity industries, has revealed that, in this industry: (i) a substantial fraction of consumers are 'switchers' in the sense that they constantly seek out for the best deal in the market; and (ii) suppliers are well aware of these consumers' dynamics and do take them into account in their pricing decisions. In particular, "companies charge more to existing ("sticky") customers whilst maintaining competitiveness in more price sensitive segments of the market.

<sup>2</sup>Chen (2005), Fudenberg and Villas-Boas (2007) and Esteves (2009b) present updated literature surveys on BBPD.

<sup>3</sup>For mobile services a PAC code is required only when the consumer wants to keep his existing telephone number when switching to the new provider.

Therefore, apart from being able to know whether or not a consumer purchased from a rival before, firms can have the tools to price discriminate between different types of old customers—those disclosing a desire to switch (called active consumers) and those showing no intention to switch (called passive consumers). Empowered with this additional information firms can have the last word over their competitors’ poaching offers. The consumer’s request of a code discloses information about his willingness to switch and gives firms an incentive to use retention offers targeted to customers who are at a risk of switching. Theoretically firms can use diverse forms of retention offers—price discounts, price matching, upgrade of services—as a way to make it less attractive for a customer to switch to a competing firm. However, according to the Ofcom report (2010, p.82) retention activity in the UK communications industry is generally in the form of a price discount.

The ability of firms to employ retention strategies will make it more difficult for firms to attract the rivals’ customers and will potentially raise welfare and antitrust concerns. Some interesting issues are the following. What is the likely impact of retention on competition and consumers? Do firms charge “excessive prices” to passive consumers? Does BBPD with retention offers enhance the dominance of the firm with a higher customer base? Who benefits and who loses when firms engage in BBPD with retention offers? Should these business practices be banned?

Despite the crucial importance of these issues, the answer to these and other related questions is not yet known. This paper takes a first step in investigating the competitive and welfare effects of retention offers in markets where firms engage in BBPD. The paper considers a two-period model with two horizontally differentiated firms competing for consumers with stable exogenous brand preferences across the two periods. These preferences are specified in the Hotelling-style linear market of unit length with firms positioned at the endpoints. Firms cannot commit to future prices. In the first-period firms charge a uniform price. In the second-period there are two stages. In the first stage, firms use the consumers’ first period purchase history to draw inferences about their preferences and price accordingly. Each firm simultaneously chooses a price to its old customers and to the rival’s previous customers. In the second stage, it is assumed that a retention discount is targeted at consumers expressing an intention to leave and is enabled by a switching process in which a provider is made aware of a customer’s intention to switch before the switching takes place (LPL process).

In order to investigate effects of retention offers when firms also employ BBPD, I first present the benchmark case where BBPD is permitted but retention offers are not allowed, either because they are not permitted or because firms cannot recognize the customers

who are at risk of switching. This benchmark is useful to understand the competitive and welfare implications of BBPD in markets operating under different switching regimes. With regard to the communications sector, the Ofcom report states that an alternative to the LPL switching regime, also in place in the UK, is the Gaining Provider Led (GPL) process which applies, for instance, to switching fixed telephony lines. Under the GPL regime, the consumer agrees a deal with the new provider before the losing provider is informed that the switch is in process. In contrast to the LPL regime, the GPL switching process does not allow firms to target counter-offers to consumers willing to switch because by the time the existing provider becomes aware of the consumer's intention to switch, the consumer has already signed the contract with a competitor.

The second-period static analysis sheds some light on the price effects of BBPD with retention counter-offers given an inherit market share. I show that firms will only engage in BBPD with retention offers when their customer base is above a threshold, i.e., when it is higher than 33%. The analysis also sheds light on whether or not BBPD with retention strategies can help a dominant firm (with a market share above 50%) to maintain its dominance. The model predicts that when BBPD is permitted but retention offers are not allowed, the dominant firm will lose its dominance under BBPD. A similar result is obtained in Gehrig *et al.* (2013). In contrast, if BBPD and retention offers are both permitted the model predicts that when the dominant firm is big enough, i.e., with a market share above 75%, although BBPD with retention activity reduces its dominance the firm can still maintain the dominant position (i.e., a market share above 50%).

While the static analysis is a useful tool, the dynamic analysis is the most appropriate to advise competition authorities. The paper shows that BBPD with retention offers gives rise to new dynamic effects. While under BBPD with no retention, the first-period equilibrium price is above the non-discrimination level, the reverse happens under BBPD with retention discounts. Regarding the second-period equilibrium prices, compared to a GPL regime with BBPD and no retention prices, the model predicts that the LPL regime with BBPD and retention leads to higher prices to all consumers that do not switch, be they passive consumers or consumers that were successfully retained after they expressed a desire to switch by requesting a code.

The welfare analysis shows that industry profits are lower and consumers' surplus and welfare are higher under the LPL regime with BBPD and retention offers than under the GPL with BBPD and no retention. The reason is that the lower second period prices for those consumers that switch and the decrease in the first-period price for all consumers more than compensate the higher prices for those consumers that do not switch. Retention offers boost welfare because the number of consumers who switch away from their preferred

product is lower than in the case where retention activity is absent.

This paper is related to the literature on competitive price discrimination,<sup>4</sup> especially the literature on behavior-based price discrimination.<sup>5</sup> Like other forms of price discrimination, BBPD can raise competition and welfare concerns. While in the switching cost approach purchase history discloses information about exogenous switching costs (e.g. Chen (1997) and Taylor (2003)), in the brand preference approach purchase history discloses information about a consumer's exogenous brand preference for a firm (e.g. Villas-Boas (1999), Fudenberg and Tirole (2000)). A common finding in this literature is that BBPD tends to intensify competition, potentially benefit consumers and reduce profits in duopoly models where (i) the market exhibits best response asymmetry,<sup>6</sup> (ii) firms are symmetric and (iii) both have information to price discriminate (e.g. Chen (1997), Villas-Boas (1999), Fudenberg and Tirole (2000), Taylor (2003) and Esteves (2010)). There are, however, some models where firms can benefit from BBPD. This can be the case when firms are asymmetric (e.g. Shaffer and Zhang (2000)), when firms' targetability is imperfect and asymmetric (Chen *et al.* (2001)) and when only one of the two firms has information to price discriminate (Chen and Zhang (2009) and Esteves (2009)). Finally, the paper is related to Gehrig *et al.* (2012) who investigate the effects of BBPD in a static asymmetric duopoly model, where one of the firms is assumed to have an inherited dominant market position (market share larger than 50%). They show that uniform pricing is a more powerful instrument than BBPD for the dominant firm to defend its market share advantage.<sup>7</sup>

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 presents the benchmark case where firms can employ BBPD but retention offers are not allowed. Section 4 presents the equilibrium analysis and Section 5 provides the welfare analysis. Section 6 concludes and the appendix collects the proofs that were omitted from the text.

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<sup>4</sup>Comprehensive surveys on competitive price discrimination are presented by Armstrong (2006) and Stole (2007).

<sup>5</sup>Chen (2005), Fudenberg and Villas-Boas (2007) and Esteves (2009b) present updated literature surveys on BBPD.

<sup>6</sup>Following Corts (1998), the market exhibits best response asymmetry when one firm's "strong" market is the other's "weak" market.

<sup>7</sup>For other recent papers on BBPD and customer recognition see also Chen and Percy (2010), Esteves and Vasconcelos (2014), Esteves and Reggiani (2014), Gehrig *et al.* (2011), (2012), Shy and Stenbacka (2013).

## 2 Model

Two firms A and B produce at zero marginal cost nondurable goods A and B.<sup>8</sup> There are two periods, 1 and 2. The total number of consumers in the market is normalized to one. In each period, each consumer wishes to buy a single unit from either firm A or B and is willing to pay at most  $v$ . The reservation value  $v$  is sufficiently high so that nobody stays out of the market. Like in Fudenberg and Tirole (2000) consumers have exogenous preferences for brands that are present from the start. Consumer preferences are specified in the Hotelling-style linear market of unit length with firms positioned at the endpoints. A consumer brand preference,  $\theta$ , is uniformly distributed on  $[0, 1]$  and is fixed over the two periods of consumption.<sup>9</sup> A consumer located at  $\theta$  incurs total cost  $p_A + t\theta$  when buying from firm A at price  $p_A$ , and incurs total cost  $p_B + t(1 - \theta)$  when buying from B at price  $p_B$ . In the brand preference approach  $t > 0$  measures how much a consumer dislikes buying a less preferred brand.

Firms are not able to observe the brand preference of individual consumers. However, in the second-period, each firm can use the information about consumers' first period purchase decisions to infer whether they prefer its brand or the rival's one and price accordingly. In the first-period price discrimination is not feasible, therefore each firm sets a single price. Suppose that at any pair of first-period prices such that all consumers purchase and both firms have positive sales, there will be a first-period cutoff  $\theta_1$  such that all consumers on the interval  $[0, \theta_1]$  buy from A and all consumers on the interval  $[\theta_1, 1]$  buy from firm B. When firms cannot commit to future prices, in the second period, each firm will offer one price to its own past customers and a different one to those who purchased from their rival before (or, new customers).<sup>10</sup>

Now I extend the body of the literature on BBPD by assuming that in the second-period there is a two-stage competition game. Like in the extant models, in the first stage, each firm simultaneously chooses a price to its own past customers,  $p_i^o$ , and a price to the new customers,  $p_i^n$ ,  $i = A, B$ . After consumers have observed its current supplier price,  $p_i^o$ , and the new supplier price,  $p_j^n$ , some of them might be willing to switch. As aforementioned, under a LPL switching regime, consumers with an intention to switch

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<sup>8</sup>The assumption of zero marginal costs can be relaxed without altering the basic nature of the results derived throughout the model.

<sup>9</sup>For a model of BBPD with imperfect correlated preferences across periods see Chen and Percy (2010).

<sup>10</sup>Because we are assuming that all consumers buy in period 1 and that no new customers enter the market in period 2, a customer who bought from firm  $j$  in period 1 is a new customer to firm  $i$  in period 2.



must go through a validation process with its existing supplier, a proof of which must be provided to the new supplier in order to complete the switch. This creates an opportunity for firms to segment their customers base between those who are willing to switch and those who are not, and try to retain the first group of consumers before they can sign any agreement with a competitor. Retention strategies are used as an attempt to make it less attractive for a customer to switch to a rival firm. Although firms may use diverse forms of retention offers—e.g., price discounts, price matching, upgrade of services—this paper focuses only on retention activity in the form of a fixed price discount targeted to a consumer expressing an intention to switch. Thus, in the second stage, it is assumed that each firm offers each customer disclosing an intention to leave (e.g, by requesting a PAC or MAC)<sup>11</sup> a secret fixed discount named  $d_i$ . Moreover, it is assumed that consumers do not bluff, i.e., only those consumers with economic reasons to switch will disclose their willingness to switch to the current supplier. Firms and consumers use the same discount factor  $\delta$ .

### 3 BBPD with no retention

Consider first the benchmark in which BBPD is permitted but retention offers are not allowed in period 2. This may occur either because the switching process in place does not allow firms to distinguish, in their base of previous customers, those who are looking to switch (active consumers) and those who are not (passive consumers); or because retention offers are not permitted. Throughout the analysis, it is considered that active customers are those who show an intention to switch by requesting, for instance, a code to complete the switching; while passive customers are those who are not willing to switch (with strong preferences for a given firm).

As mentioned in the Introduction, with regard to the UK communications sector, an alternative to the losing provider led (LPL) switching regime, is the gaining provider led (GPL) process which applies, for instance, to switching fixed telephony lines. In contrast to the LPL, the GPL regime does not allow firms to make counteroffers to consumers willing to switch because by the time the existing provider becomes aware of the consumer’s intention to switch, the consumer has already signed the contract with a competitor.

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<sup>11</sup>In the UK consumers wishing to switch their mobile telephony services must contact their existing provider and request a Porting Authorisation Code (PAC) which they must communicate to the new provider in order to complete the switch. The same procedure is applied for switching broadband services, in which case the required code is the Migrations Authorisation Code (MAC).

This benchmark is useful to advise competition policy agencies with regard to (i) the competitive and welfare implications of BBPD in markets operating under different switching regimes and (ii) the likely impact of retention offers on prices, profits and consumer welfare in comparison to the case where this practice is forbidden or not feasible.

The analysis of BBPD with no retention strategies (under the GPL switching regime) is based on the simplified version of Fudenberg and Tirole (2000) with consumer preferences uniformly distributed on the interval  $[0, 1]$ .

Let the superscript  $NR$  denote no retention.

**Proposition 1.** *When firms can price discriminate between old and new customers but retention offers are not allowed, second period equilibrium prices are:*

(i) *If  $\theta_1 \leq \frac{1}{4}$ :*

$$p_A^{o,NR} = t(1 - 2\theta_1); p_A^{n,NR} = \frac{1}{3}t(3 - 4\theta_1)$$

$$p_B^{o,NR} = \frac{1}{3}t(3 - 2\theta_1); p_B^{n,NR} = 0.$$

(ii) *If  $\frac{1}{4} \leq \theta_1 \leq \frac{3}{4}$ :*

$$p_A^{o,NR} = \frac{1}{3}t(2\theta_1 + 1); p_A^{n,NR} = \frac{1}{3}t(3 - 4\theta_1),$$

$$p_B^{o,NR} = \frac{1}{3}t(3 - 2\theta_1); p_B^{n,NR} = \frac{1}{3}t(4\theta_1 - 1).$$

(iii) *If  $\theta_1 \geq \frac{3}{4}$ :*

$$p_A^{o,NR} = \frac{1}{3}t(2\theta_1 + 1); p_A^{n,NR} = 0,$$

$$p_B^{o,NR} = t(2\theta_1 - 1); p_B^{n,NR} = \frac{1}{3}t(4\theta_1 - 1).$$

**Proof.** See the Appendix.

In order to shed some light about the impact of BBPD when firms depart with an inherited exogenous base of customers, let  $s_i^{2,NR}$ ,  $i = A, B$ , denote firm  $i$  second period market share (with BBPD and no retention) given firm A's inherited market share equal to  $\theta_1$ .

**Corollary 1.** *When  $\theta_1 = \frac{1}{2}$ , firms will share equally the market in period 2. However, if  $\theta_1 > \frac{1}{2}$  then  $s_A^{2,NR} < \frac{1}{2}$  and  $s_B^{2,NR} > \frac{1}{2}$ . The reverse happens when  $\theta_1 < \frac{1}{2}$ .*

**Proof.** See the Appendix.

The previous corollary shows that if firms depart with an equal base of customers, they will share equally the market in period 2. In contrast, if firms depart with asymmetric inherited market shares, then BBPD with no retention destroys the dominance of the larger firm. Specifically, the smaller firm will become the leader while the larger firm will become the smaller one. A similar result is obtained in Gehrig *et al.* (2012), who conclude that uniform pricing is a more powerful instrument than BBPD for the dominant firm to defend its market share advantage. This static analysis will be useful to draw some conclusion about the effects of BBPD in an industry where one of the firms can use retention offers to defend its dominance.

Now look at first-period price competition. Let  $p_i^1$  represent firm  $i$ 's first-period price,  $i = A, B$ . Following Fudenberg and Tirole (2000) we can establish the following proposition.

**Proposition 2.** *When BBPD is permitted and retention is not allowed, there is a symmetric subgame perfect nash equilibrium in which:*

(i) *First-period equilibrium prices are  $p^{1,NR} = t \left(1 + \frac{\delta}{3}\right)$  and the first-period market is split symmetrically with  $\theta^{1,NR} = \frac{1}{2}$ .*

(ii) *Second-period equilibrium price for old and new customers are, respectively,  $p^{o,NR} = \frac{2}{3}t$  and  $p^{n,NR} = \frac{1}{3}t$ .*

(iii) *In period 2, consumers on the intervals  $\left[0, \frac{1}{3}\right]$  and  $\left[\frac{2}{3}, 1\right]$  do not switch and consumers on the interval  $\left[\frac{1}{3}, \frac{2}{3}\right]$  switch to a new supplier.*

(iv) *Each firm's overall profit is equal to  $\pi^{NR} = \frac{t(9+8\delta)}{18}$ .*

## 4 BBPD with retention strategies

As aforementioned I now assume that in the second period apart from being able to distinguish their own previous customers and those who bought from the rival before (new customers), firms have the tool to recognize, in their base of previous customers, those who are at risk of switching. Firms can now use retention offers as an attempt to make it less attractive for a customer to switch to a competing firm. We look at retention activity in the form of a price discount offered to a consumer showing an intention to switch. The use of a retention price discount is also a form of price discrimination based on consumers' behavior (in this case the request of a code to complete the switch). This form of retention activity is not price discrimination between old and new customers

(as in the existing models of BBPD), but rather between different types of a firm's old consumers (those who are willing to switch and those who are not).

With that in mind the game is solved working backward from the second period.

## 4.1 Second-period

Suppose that the first period prices lead to a cutoff  $\theta_1 \in [0, 1]$  such that a consumer located at  $\theta_1$  is indifferent between buying from A and B in period 1. With no loss of generality, look at firm A's turf on  $[0, \theta_1]$ . Some of firm A's first-period consumers might be willing to switch to B given the observed second period prices  $\{p_A^o, p_B^n\}$ . Under a LPL switching regime, these consumers will need to contact the current provider (firm A) and request a code (e.g., PAC or MAC) to complete the switching process to firm B. It is the request of this code that allows firm A to get back to them with a secret retention price discount  $d_A$ . Thus, in the second-stage of period 2, the indifferent consumer between staying with A after being exposed to a retention campaign and switching to B is located at  $\theta_A$ , such that

$$\theta_A = \frac{1}{2} + \frac{p_B^n - p_A^o + d_A}{2t}. \quad (1)$$

In the group of firm A's own customers, the indifferent consumer between being passive and active is located at  $\theta_A^c$  such that

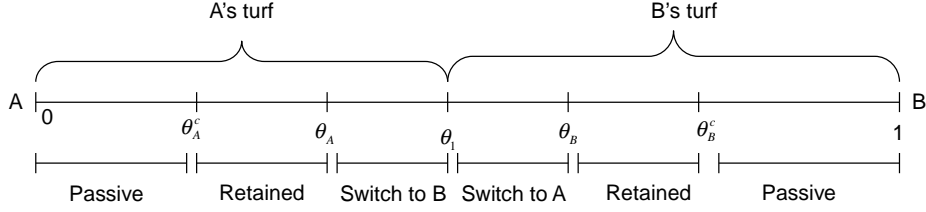
$$\theta_A^c = \frac{1}{2} + \frac{p_B^n - p_A^o}{2t}.$$

A similar reasoning is applied to derive the location of the indifferent consumer between being passive and active, namely  $\theta_B^c$  in the group of customers who bought from B in period 1, those on the interval  $[\theta_1, 1]$ .

Given the existence of a first-period cutoff, the second-period situation is as depicted in Figure 1: Consumers to the left of  $\theta_1$  lie in firm A's turf and those to the right lie in firm B's. On firm A's turf part consumers to the left of  $\theta_A^c$  are passive, active consumers on the interval  $[\theta_A^c, \theta_1]$  are retained (those located at the interval  $[\theta_A^c, \theta_1]$ ); while those on the interval  $[\theta_A, \theta_1]$  do in fact switch to firm B even though they have been exposed to a retention offer.

In the second stage firm A and B solve, respectively, the following problem:

$$\text{Max}_{d_A} (p_A^o - d_A) (\theta_A - \theta_A^c), \quad (2)$$



$$\underset{d_B}{Max} (p_B^o - d_B) (\theta_B^c - \theta_B). \quad (3)$$

It is straightforward to obtain that the price discount offered by firm  $A$  and  $B$  is, respectively,  $d_A = \frac{p_A^o}{2}$  and  $d_B = \frac{p_B^o}{2}$ . With no loss of generality look on firm  $A$ 's turf. In the first-stage of period 2, firm  $A$  and  $B$  solve respectively:

$$\underset{p_A^o}{Max} \pi_A^o = p_A^o \theta_A^c + (p_A^o - d_A)(\theta_A - \theta_A^c),$$

$$\underset{p_B^n}{Max} \pi_B^n = p_B^n (\theta_1 - \theta_A).$$

A similar reasoning is applied to firm  $B$ 's turf on the interval  $[\theta_1, 1]$ .

**Proposition 3.** *When firms can employ BBPD and retention offers the second period equilibrium prices and profits are:*

(i) If  $\theta_1 \leq \frac{1}{3}$ :

$$p_A^o = t(1 - 2\theta_1); \quad p_A^n = \frac{2t}{5}(2 - 3\theta_1)$$

$$p_B^o = \frac{2t}{5}(3 - 2\theta_1); \quad d_B = \frac{t}{5}(3 - 2\theta_1); \quad p_B^n = 0.$$

$$\pi_A^2 = t(1 - 2\theta_1)\theta_1 + \frac{2t}{25}(3\theta_1 - 2)^2 \quad (4)$$

$$\pi_B^2 = \frac{3t}{50}(2\theta_1 - 3)^2 \quad (5)$$

(ii) If  $\frac{1}{3} \leq \theta_1 \leq \frac{2}{3}$ :

$$p_A^o = \frac{2t}{5}(2\theta_1 + 1); \quad d_A = \frac{t}{5}(2\theta_1 + 1); \quad p_A^n = \frac{2t}{5}(2 - 3\theta_1)$$

$$p_B^o = \frac{2t}{5}(3 - 2\theta_1); \quad d_B = \frac{t}{5}(3 - 2\theta_1); \quad p_B^n = \frac{2t}{5}(3\theta_1 - 1).$$

$$\pi_A^2 = \frac{t}{50}(48\theta_1^2 - 36\theta_1 + 19) \quad (6)$$

$$\pi_B^2 = \frac{t}{50} (48\theta_1^2 - 60\theta_1 + 31) \quad (7)$$

(iii) If  $\theta_1 \geq \frac{2}{3}$ :

$$p_A^o = \frac{2t}{5} (2\theta_1 + 1); \quad d_A = \frac{t}{5} (2\theta_1 + 1); \quad p_A^n = 0$$

$$p_B^o = t(2\theta_1 - 1); \quad p_B^n = \frac{2t}{5} (3\theta_1 - 1).$$

$$\pi_A^2 = \frac{3t}{50} (2\theta_1 + 1)^2 \quad (8)$$

$$\pi_B^2 = (1 - \theta_1) t(2\theta_1 - 1) + \frac{2}{25} t (3\theta_1 - 1)^2 \quad (9)$$

**Proof.** See the Appendix.

It is interesting to note that a firms will only employ a retention strategy if its customer base is above a threshold. Considering, for instance, the case of firm A, we observe that it will only offer a retention discount its first period market share is larger than  $\frac{1}{3}$ . It is also interesting compare the equilibrium retention discount obtained—50% of the second period current price to old customers—, with existing empirical evidence. Considering the Ofcom report (2010, p.82), we find that retention discounts generally vary between 32% and 60% of the current price in mobile telephony and between 25% and 44% of the current price in broadband services.

Before proceeding to the analysis of competition in period 1, we next try to draw some conclusions about the competitive effects of BBPD with and without retention in an industry where firms would depart with an inherited exogenous market share. With no loss of generality consider the case of firm A. From Proposition 1 and Proposition 3 it is straightforward to obtain the following result.

**Corollary 2.** *Comparing the second-period prices of BBPD with and without retention strategies:*

- (i) *firm A's passive consumers pay higher prices with retention offers when  $\theta_1 > \frac{1}{3}$ .*
- (ii) *firm A's retained consumers pay higher prices when  $\theta_1 < \frac{4}{7}$ , while they pay a lower price when  $\theta_1 > \frac{4}{7}$ .*
- (iii) *firm A's price to new customers with retention strategies is always below its counterpart when this activity is banned.*

Figure 2 illustrates the second-period equilibrium prices given an inherited market share with and without retention strategies in firm A's turf A assuming that  $t = 1$ .

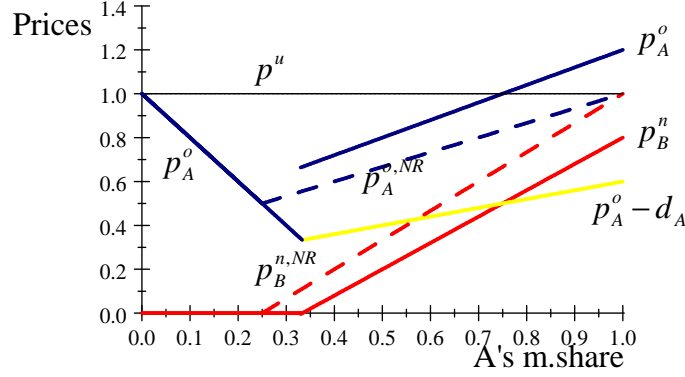


Figure 1: Second-period prices with and without retention strategies

Corollary 2 suggests that it is important to take into account whether one of the firms has a dominant position in the market when trying to understand the competitive effects of retention offers. Considering for instance the case of firm A, we observe that its existing consumers pay the same price with and without retention activity when firm A has a smaller market share, specifically when  $\theta_1 \leq \frac{1}{4}$ . At an interior solution (not too strong asymmetry between firms), we observe that because firms are able to segment their existing customer base between “active” and “passive” they can charge a much higher price to passive consumers than if retention activity were banned ( $p_A^o > p_A^{o,NR}$ ). As said, note that firms will only try to retain customers when their customer base is above a threshold. Firm A, for instance, will only offer retention discounts if  $\theta_1 > \frac{1}{3}$ . When we move from BBPD with no retention to BBPD with retention we find that retained customers are charged a higher price when  $\theta_1 < \frac{4}{7}$ , while they face a lower price when  $\theta_1 > \frac{4}{7}$ . The intuition is the following. When  $\theta_1 > \frac{1}{2}$  some consumers in firm A’s turf are B-oriented consumers, thus firm A needs to price more aggressively if it wants to avoid switching. Regarding firm A’s price to new customers ( $p_A^n$ ) we find that given firm B’s retention offers, firm A will need to be more aggressive with their headline price offers (prices for new customers) if it wants to convince customers to switch. Thus, the poaching price ( $p^n$ ) with retention offers is always below its counterpart when retention is absent.

**Corollary 3.** *When firms have symmetric initial market shares they split equally the market in the second-period. When  $\theta_1 \in [\frac{1}{3}, \frac{2}{3}]$  BBPD with retention strategies leads the dominant firm to lose its dominance, that is  $s_A^2 \leq \frac{1}{2}$  ( $s_B^2 \geq \frac{1}{2}$ ) if  $\theta_1 \geq \frac{1}{2}$  ( $\theta_1 \leq \frac{1}{2}$ ). In contrast, the bigger firm is able to maintain its dominance when the asymmetry in the market is strong enough. Particularly, it follows that  $s_A^2 \geq \frac{1}{2}$  ( $s_B^2 \leq \frac{1}{2}$ ) if  $\theta_1 \in [\frac{3}{4}, 1]$  and*

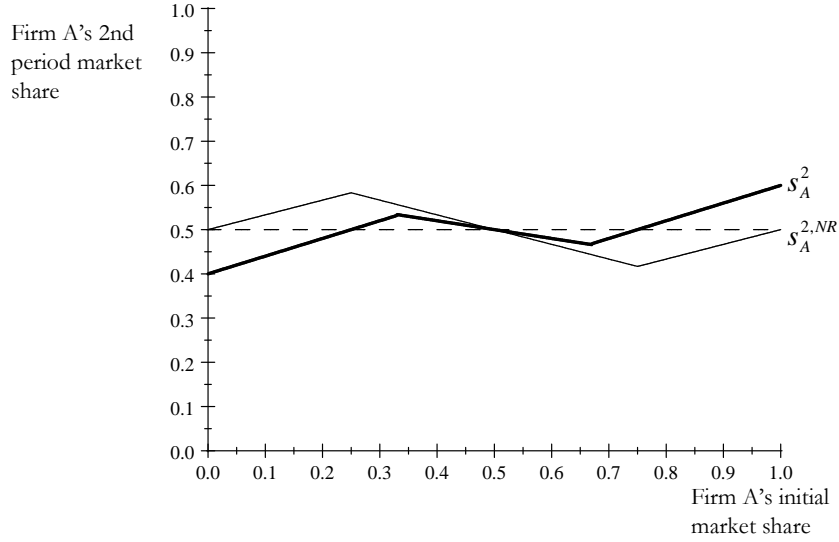


Figure 2: Firm A's second-period market share

$$s_A^2 \leq \frac{1}{2} \quad (s_B^2 \geq \frac{1}{2}) \quad \text{if } \theta_1 \in [0, \frac{1}{4}].$$

**Proof.** See the Appendix.

Figure 3 plots firm A's second period market share when it departs with an exogenous inherited market share in the benchmark case of BBPD with no retention strategies  $s_A^{2,NR}$  and in the case of BBPD with retention discounts,  $s_A^2$ . It confirms the findings in Corollary 1 and 3. As seen before the bigger firm (initial market share higher than 50%) will always lose its dominance under BBPD and no retention. Note also that for any  $\theta_1 \geq 0.5$ , it is always the case that  $s_A^2 \geq s_A^{2,NR}$ .

In contrast, it is important to stress that when firms can try to retain their previous clientele, BBPD can help the dominant firm to maintain its dominance, i.e., BBPD may not destroy the dominance of the bigger firm. This happens when the initial market share of the bigger firm is sufficiently high (i.e., higher than 75% of the market). If, for instance, firm A departs with an initial market share of 90%, BBPD with retention activity will reduce its second-period market share to 56%.

Figure 4 plots both firms' second-period profits as a function of  $\theta_1$ . It gives some insight about the profit effects of retention offers if firms had initial asymmetric customer bases. From a static point of view, as expected, we observe that the dominant firm earns



higher profits than the smaller firm. With no retention activity, at the interior solution ( $\frac{1}{4} \leq \theta_1 \leq \frac{3}{4}$ ), we have that both firms make the same profit in the second period. With retention strategies this is no longer the case because each firm's profit increases with its own initial market share. At the interior solution ( $\frac{1}{3} \leq \theta_1 \leq \frac{2}{3}$ ) both firms earn the same profit only when they are initially symmetric. For this reason when BBPD with retention discounts are permitted, each firm has a strategic incentive to build up its first-period market share.

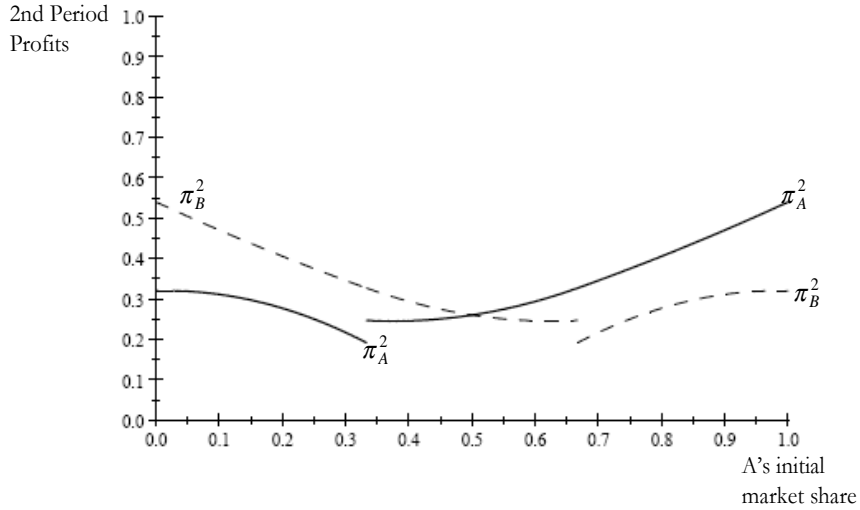


Figure 3: Second-period profits

## 4.2 First-period

Next we look at the equilibrium first-period pricing and consumption decisions. Because firms are forward looking they rationally anticipate how today's price decision will affect their second-period pricing and profits. Consumers are also sophisticated in the sense that in equilibrium they correctly anticipate the firms second-period price discrimination strategies.

Let firm A's first-period price be  $p_A^1$  and firm B's first-period price be  $p_B^1$ . The marginal consumer in the first period will surely switch in the second period to take advantage of the poaching price. If first-period prices lead to a cutoff  $\theta_1$ , the consumer located at  $\theta_1$  is indifferent between buying from firm A in period 1 at price  $p_A^1$  and then buying from firm B in period 2 at the poaching price  $p_B^2$ , or buying from firm B in period 1 at price

$p_B^1$  and then buying from firm A at the poaching price  $p_A^n$ . At an interior solution,

$$p_A^1 + t\theta_1 + \delta(p_B^n + t(1 - \theta_1)) = p_B^1 + (1 - \theta_1)t + \delta(p_A^n + t\theta_1),$$

thus

$$\theta_1 = \frac{t + p_B^1 - p_A^1 + \delta[p_A^n(\theta_1) - p_B^n(\theta_1)]}{2t(1 - \delta)}.$$

Using the expressions for  $p_A^n$  and  $p_B^n$  defined in Proposition 3 it follows that

$$\theta_1 = \frac{1}{2} + \frac{5(p_B^1 - p_A^1)}{2t(\delta + 5)}. \quad (10)$$

If price discrimination is not permitted or if  $\delta = 0$ , then  $\frac{\partial\theta_1}{\partial p_A^1} = -\frac{1}{2t}$ . Under BBPD with retention strategies we have

$$\frac{\partial\theta_1}{\partial p_A^1} = -\frac{1}{2t\left(\frac{\delta}{5} + 1\right)}, \quad (11)$$

while under BBPD with no retention strategies we have

$$\frac{\partial\theta_1}{\partial p_A^1} = -\frac{1}{2t\left(\frac{\delta}{3} + 1\right)}.$$

Thus, as long as  $\delta > 0$ , with BBPD consumers react less to price reductions in the first period than they would in a static model of this kind. Additionally, it is straightforward to see that demand will be less elastic in the first period if BBPD is permitted but retention offers are not allowed.

Now consider the equilibrium choices of  $p_A^1$  and  $p_B^1$ . At an interior solution, firm A and B's overall objective function is, respectively, given by

$$p_A^1 \left( \frac{1}{2} + \frac{5(p_B^1 - p_A^1)}{2t(\delta + 5)} \right) + \delta \left( \frac{1}{50}t \left( 48(\theta_1(p_A^1, p_B^1))^2 - 36\theta_1(p_A^1, p_B^1) + 19 \right) \right), \quad (12)$$

$$p_B^1 \left( \frac{1}{2} + \frac{5(p_A^1 - p_B^1)}{2t(\delta + 5)} \right) + \delta \left( \frac{1}{50}t \left( 48(\theta_1(p_A^1, p_B^1))^2 - 60(\theta_1(p_A^1, p_B^1)) + 31 \right) \right). \quad (13)$$

Substituting equation (10) into equations (12) and (13) it is straightforward to obtain the following proposition.

**Proposition 4.** *There is a symmetric subgame perfect nash equilibrium in which:*

(i) *first-period equilibrium prices are  $p^1 = t\left(1 - \frac{\delta}{25}\right)$  and both firms share equally the market in period 1;*

(ii) *second-period equilibrium prices are  $p^o = \frac{4}{5}t$ ,  $d = \frac{2}{5}t$  and  $p^n = \frac{1}{5}t$ .*

(iii) Consumers on the intervals  $[0, \frac{1}{5}]$  and  $[\frac{4}{5}, 1]$  show are not willing to switch, consumers on the intervals  $[\frac{1}{5}, \frac{2}{5}]$  and  $[\frac{3}{5}, \frac{4}{5}]$  show an intention to switch but are retained, and consumers on the intervals  $[\frac{2}{5}, \frac{3}{5}]$  show an intention to leave and do switch in spite of being exposed to a retention offer.

(iv) Each firm's second-period equilibrium profit is equal to  $\frac{13}{50}t$ , while the first period equilibrium profit equals  $\frac{t}{2} (1 - \frac{\delta}{25})$ . Overall equilibrium profit under BBPD with retention offers is equal to  $\frac{t}{50} (12\delta + 25)$ .

**Proof.** See the Appendix.

Proposition 4 highlights that the possibility of firm engaging in BBPD with retention offers under a LPL switching regime, leads to higher prices to all consumers that do not switch (be they consumers that are not looking to switch or consumers that were successfully retained after they expressed a desire to switch by requesting a code) compared to a GPL process. (Note  $p^o = \frac{4}{5}t > p^{o, NR} = \frac{2}{3}t$  and  $p^o - d = \frac{2}{5}t > p^{n, NR} = \frac{1}{3}t$ .) In particular, even consumers that obtain a lower price under retention end up paying a higher price than they would in the absence of retention offers under a GPL process. Regarding, the consumers that do switch we find that they actually get a lower price in a LPL switching regime with retention offers than they would in the absence of retention under a GPL regime. This is because the offer of a retention discount implies that in order to induce switching, it is necessary to offer very low prices to those consumers that are eager to switch.

Regarding, first-period prices an interesting finding is that the first-period price with BBPD and retention strategies is *below* the uniform price. This result should be compared with first-period equilibrium price *above* the uniform price when BBPD is used without retention offers. In general when firms can engage in price discrimination based on purchase history there are two effects on first-period prices: a consumer-side effect and a firm-side effect. When consumers are non myopic they correctly anticipate the second period prices, become less price sensitive in period 1 and so there is a positive effect on first-period prices. When firms are forward looking, they also take into account that a change in first-period prices will affect the second-period prices and profits.

In the benchmark case of BBPD with no retention strategies a change of first-period prices has no effect on second-period profit because with a uniform distribution a firm's marginal gains in the second-period market are exactly offset by losses in the first-period market ( $\frac{\partial \pi^2}{\partial p_1^A} = 0$ ). In this case the decrease (the price sensitivity of consumers in period 1 that occurs when we move from no discrimination to BBPD determines the result of

first-period prices above the non-discrimination level.

Looking at the effect of first-period prices on second-period profit when firms can employ BBPD with retention offers we find that in the symmetric equilibrium  $\frac{\partial \pi^2}{\partial p_A^1} = -\frac{3\delta}{5(\delta+5)} < 0$ . This suggests that each firm has a strategic incentive to enlarge its turf in period 1, which is achieved by competing more aggressively in that period. Therefore, in comparison to no-discrimination, firms charge lower first-period prices when they can compete with BBPD and retention offers because the firm-side effect is stronger than the consumer-side effect.

## 5 Welfare analysis

This section investigates the welfare effects of BBPD when firms engage in a retention activity in an attempt to make it less attractive for a customer to switch to a competing firm. We compare this possibility with two other price competition scenarios; the one where price discrimination is not permitted and the other where BBPD is permitted but retention offers are not allowed. Let the superscript *nd* denote no discrimination; *NR* denote BBPD with no retention activity and *R* denote BBPD with retention discounts. Further, let  $\pi_{ind}$  denote industry profits, *CS* denote consumer surplus and *W* denote overall welfare.

**Proposition 5.** *For any  $\delta > 0$  is is always true:*

- (i)  $\pi_{ind}^{nd} > \pi_{ind}^{NR} > \pi_{ind}^R$
- (ii)  $CS^{nd} < CS^{NR} < CS^R$
- (iii)  $W^{nd} > W^R > W^{NR}$

**Proof.** See the Appendix.

Proposition 5 highlights that in comparison to uniform pricing, price discrimination based on purchase history is bad for profits and overall welfare but good for consumers. However, conditional on BBPD being employed, the use of a retention strategy through a price discount offered to those consumers showing an intention to switch boosts consumer surplus and overall welfare at the expense of industry profits.

In order to discuss the impact of retention offers for specific groups of consumers let we compare first BBPD with no retention activity with uniform pricing. In the Fudenberg-Tirole model with the uniform, price discrimination has no effect on consumer welfare for the consumers on the interval  $[0, \frac{1}{3}]$  and  $[\frac{2}{3}, 1]$ . These consumers do not switch in equilibrium and their present value payment for the two periods of consumption is the same

in both pricing regimes, i.e.,  $(1 + \delta)t$ . In contrast, consumers on the interval  $[\frac{1}{3}, \frac{2}{3}]$  switch from one firm to another and the present value of their payment is equal to  $(1 + \delta)t - \frac{\delta t}{3}$ . In comparison to no discrimination, the group of switchers is strictly better off under BBPD with no retention strategies.

Look now at BBPD with retention offers. Consumers on the intervals  $[0, \frac{1}{5}]$  and  $[\frac{2}{5}, 1]$  do not signal an intention to switch and as a result of that they face a higher second-period price. Their present value payment for the two periods of consumption is  $(1 + \delta)t - \frac{6\delta t}{25}$ . Thus, the decrease in the first-period price more than compensates the second period loss, implying that the group of passive consumers is strictly better off with BBPD and retention offers than under no discrimination (where they pay  $(1 + \delta)t$ ). Consumers on the intervals  $[\frac{1}{5}, \frac{2}{5}]$  and  $[\frac{3}{5}, \frac{4}{5}]$  show an intention to switch but are retained. The present value of the price paid by these consumers in both periods is  $(1 + \delta)t - \frac{16\delta t}{25}$ . These consumers are also clearly better off when firms employ BBPD with retention discounts. Consumers on the intervals  $[\frac{1}{3}, \frac{2}{5}]$  and  $[\frac{3}{5}, \frac{2}{3}]$  decide not to switch when we move from BBPD alone to BBPD with retention offers. The present value of their payment is equal to  $(1 + \delta)t - \frac{6\delta t}{25}$  with retention discounts, while it is equal to  $(1 + \delta)t - \frac{\delta}{3}t$  with no retention. Consequently, this group of consumers also benefits when firms employ BBPD and retention discounts. Finally, the poached consumers on the interval  $[\frac{2}{5}, \frac{3}{5}]$  switch from one firm to another under retention strategies. The present value of the price paid by them for the two periods of consumption is equal to  $(1 + \delta)t - \frac{21\delta t}{25}$ .

Summing up, BBPD with retention strategies reduces the present value of the price paid by consumer in all segments, explaining that the use of BBPD with retention discounts under a LPL switching process can benefit consumers in comparison to the case where retention is absent under the GPL regime.

Regarding the aggregate effects on welfare, because in the present model there is no role for price discrimination to increase aggregate output, variations in welfare are uniquely explained by the “desutility” supported by those consumers who buy inefficiently.<sup>12</sup> As retention discounts are used by firms as an attempt to make it less attractive for a customer to switch to a rival firm, a smaller number of consumers do in fact switch in equilibrium. As a result of that in comparison to BBPD alone, BBPD with retention offers boosts welfare because it gives rise to less inefficient switching.

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<sup>12</sup>For a model where BBPD can affect aggregate output see Esteves and Reggiani (2014).

## 6 Conclusions

The economics literature on price discrimination by purchase history has hitherto focused on the assumption that (i) firms have only the required information to price discriminate between old and new customers and that (ii) firms have no way to react to the rivals' poaching offers. Interestingly, in some of the markets where firms often price discriminate between their own and the rivals' consumers, the switching processes currently in place in many countries have allowed firms to become aware of an existing customer's willingness to leave before the switching takes place. Consequently, firms have been increasingly able to recognize different categories of old customers—those willing to stay and those willing to switch—and try to raise the switching barriers by engaging in retention offers.

This paper has taken a first step in investigating the impact of behavior-based price discrimination in markets where firms are allowed to try to retain their previous clientele, by offering those showing an intention to switch a price discount.

In order to understand the implications of these business practice in asymmetric markets, we had looked at the static second-period analysis. It highlights that firms will only offer retention discounts if their customer base is above a threshold (i.e., above 33%). Further, the static analysis also sheds some light on whether or not BBPD with retention strategies in a LPL regime helps a dominant firm (with a market share above 50%) to maintain its dominant position. If BBPD is possible but retention activity is forbidden, the dominant firm will lose its dominance under price discrimination. In contrast, if the dominant firm is big enough (with a market share above 75%), although BBPD with retention offers makes the market more competitive it allows the bigger firm to maintain its dominance.

While the static analysis is a useful tool, the dynamic one is the most appropriate to inform competition authorities about the economic effects of BBPD with retention offers. Take into account the intertemporal effects of BBPD with retention offers, the paper shows that the first period equilibrium price with retention strategies is below its non-discrimination counterpart, which contrasts with first period price above the non-discrimination level when these business strategies are forbidden. Regarding second-period prices, the possibility of firm engaging in BBPD with retention offers under a LPL switching regime, leads to higher prices to all consumers that do not switch compared to a GPL process. In contrast, the consumers that do switch get a lower price in a LPL switching regime with retention offers than they would in the absence of retention under a GPL regime. In spite of this, we show that the present value of the price paid by consumers who do not switch is lower under BBPD with retention offers under a LPL

switching regime than when it is banned, suggesting that the higher second-period prices are more than compensated by the lower first-period price. In sum the paper shows that BBPD with retention strategies under a LPL switching regime, can reduce the present value of the price paid by consumers in all segments, compared to BBPD with no retention (GLP regime).

As in other models where consumers have stable exogenous brand preferences, in comparison to uniform pricing the instrument of BBPD is bad for profits and welfare but good for consumers. The model predicts that industry profits are lower and consumers' surplus and welfare is higher under LPL with retention offers than under GPL without retention activity.

However, it is important to stress that the results obtained in this model should be interpreted with care. Like other models of BBPD, the model has some limitations. One limitation is the unit demand assumption.<sup>13</sup> In these models, output is constant whatever the pricing policy (discriminatory or uniform) and the price levels. Prices only affect how the total surplus available in the economy is shared between consumers and firms. A pricing policy that generates more switching will yield a lower welfare. As the present model predicts that the present value of the price paid by all consumer segments decreases with retention activity, extending the model by relaxing the unit demand assumption would produce the same qualitative welfare results. Another limitation is the assumption of preferences uniformly distributed. Extending the model to other distribution of consumer preferences would produce insights about the effects of BBPD and retention offers in markets characterized by a large tail of consumers with preferences for one of the firms and a small tail of consumers with preferences for the other firm. It is likely in this scenario industry profits may be higher and consumers' surplus may be lower under retention offers. It is therefore important to get a better understanding of brand loyalty and consumer inertia, in the markets under consideration if we are to gain a better understanding of the distribution of consumers' preference.

Finally, this model assumed that firms offer the same discount to all consumers expressing an intention to leave. In practice, firms offer different discounts to consumers and these may be the outcome of a "bargaining process" which may be influenced by the consumer's level of brand loyalty.

Notwithstanding the model addressed in this paper is far from covering all complex aspects of real markets, it has tried to offer a closer approximation of reality where firms have increasingly more consumer information to react to the rivals' poaching offers. Al-

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<sup>13</sup>It would be interesting to explore retention offers in Esteves and Regiani (2014) framework.

though any advice to a policy agency should take into account the features of each market, in those markets that could be reasonably well represented by the features of the current model, restrictions on the ability of firms to employ retention offers through under a LPL switching process would benefit industry profits at the expense of consumer welfare.

## A Proofs

Some of the proofs in this technical appendix need to be improved.

**Proof of Proposition 1.** Consider second-period competition in firm A's first period customer base  $[0, \theta_1]$ . Let  $p_A^o$  represent firm A's price to its previous customers and  $p_B^n$  firm B's poaching price.

The indifferent consumer between buying again from A at price  $p_A^o$  and switching to B and pay  $p_B^n$  is located at  $\theta_A$  such that

$$\begin{aligned} p_A^o + t\theta_A &= p_B^n + t(1 - \theta_A) \\ \theta_A &= \frac{1}{2} + \frac{p_B^n - p_A^o}{2t}. \end{aligned} \quad (14)$$

This implies that at prices  $p_A^o, p_B^n$ , consumers on the interval  $[0, \theta_A]$  have a strong preference from A and buy again product A. Differently, consumers on the interval  $[\theta_A, \theta_1]$  switch from A to B. Using similar arguments it is straightforward to show that in B's turf the indifferent consumer between staying with B and switching to A is located at

$$\theta_B = \frac{1}{2} + \frac{p_B^o - p_A^n}{2t}. \quad (15)$$

Thus, consumers on the interval  $[\theta_1, \theta_B]$  switch from B to A and consumers on the interval  $[\theta_B, 1]$  buy again from B. In A's turf, each firm solves the following problem

$$\begin{aligned} &Max_{p_A^o} \left\{ p_A^o \left( \frac{1}{2} + \frac{p_B^n - p_A^o}{2t} \right) \right\}, \\ &Max_{p_B^n} \left\{ p_B^n \left( \theta_1 - \frac{1}{2} - \frac{p_B^n - p_A^o}{2t} \right) \right\}. \end{aligned}$$

Firm A's best response is

$$p_A^o = \frac{1}{2}t + \frac{1}{2}p_B^n$$

and firm B's best response is

$$p_B^n = \frac{1}{2}p_A^o - \frac{1}{2}t + t\theta_1.$$



It thus follows that

$$\begin{aligned} p_A^o &= \frac{1}{3}t(2\theta_1 + 1) \\ p_B^n &= \frac{1}{3}t(4\theta_1 - 1). \end{aligned}$$

It is straightforward to obtain that the equilibrium prices in turf B are

$$\begin{aligned} p_B^o &= \frac{1}{3}t(3 - 2\theta_1) \\ p_A^n &= \frac{1}{3}t(3 - 4\theta_1). \end{aligned}$$

Note however that it is a dominated strategy for each firm to quote a poaching price below the marginal cost, which in this case is equal to zero. From  $p_B^n \geq 0$  it must be true that  $\theta_1 \geq \frac{1}{4}$ . Otherwise, i.e., when  $\theta_1 \leq \frac{1}{4}$  it follows that  $p_B^n = 0$ , and so firm A's best response in order not to lose the marginal consumer located at  $\theta_1$  is to quote  $p_A^o + t\theta_1 = t(1 - \theta_1)$ , from which we obtain  $p_A^o = t(1 - 2\theta_1)$ . Thus, when  $\theta_1 \leq \frac{1}{4}$  second-period equilibrium prices are

$$p_A^o = t(1 - 2\theta_1); p_A^n = \frac{1}{3}t(3 - 4\theta_1), \quad (16)$$

$$p_B^o = \frac{1}{3}t(3 - 2\theta_1); p_B^n = 0. \quad (17)$$

Similarly it is straightforward to find that if  $\theta_1 \geq \frac{3}{4}$

$$p_A^o = \frac{1}{3}t(2\theta_1 + 1); p_A^n = 0$$

$$p_B^o = t(2\theta_1 - 1); p_B^n = \frac{1}{3}t(4\theta_1 - 1).$$

This completes the proof. ■

**Proof of Corollary 1.** From these second-period equilibrium prices it is easy to obtain that each firm second-period market share,  $s_A^2$  and  $s_B^2$ . At the interior solution  $\theta_1 \in [\frac{1}{4}, \frac{3}{4}]$

$$s_A^2 = \frac{2 - \theta_1}{3} \text{ and } s_B^2 = \frac{1 + \theta_1}{3},$$

When  $\theta_1 \in [0, \frac{1}{4}]$

$$s_A^2 = \frac{2\theta_1 + 3}{6} \text{ and } s_B^2 = \frac{3 - 2\theta_1}{6},$$

When  $\theta_1 \in [\frac{3}{4}, 1]$

$$s_A^2 = \frac{1}{6}(2\theta_1 + 1) \text{ and } s_B^2 = \frac{1}{6}(5 - 2\theta_1).$$

Straightforward computations prove that when  $\theta_1 \in [\frac{1}{4}, \frac{3}{4}]$ ,  $s_A^2 > \frac{1}{2}$  ( $s_B^2 < \frac{1}{2}$ ) iff  $\theta_1 < \frac{1}{2}$  while  $s_A^2 < \frac{1}{2}$  ( $s_B^2 < \frac{1}{2}$ ) iff  $\theta_1 > \frac{1}{2}$ . On the interval  $\theta_1 \in [0, \frac{1}{4}]$ ,  $s_A^2 > \frac{1}{2}$  ( $s_B^2 < \frac{1}{2}$ ) iff  $\theta_1 > 0$ , which is always true. Finally, when  $\theta_1 \in [\frac{3}{4}, 1]$  it follows that  $s_A^2 < \frac{1}{2}$  ( $s_B^2 > \frac{1}{2}$ ) iff  $\theta_1 < 1$  which is always true. ■

**Proof of Proposition 3.** Look first at firm A's turf. Given that in the second stage of period 2 firm A offers a discount  $d_A = \frac{p_A^o}{2}$  to consumers showing an intention to leave firm B anticipates this Behavior and solves the following problem in the first stage of period 2:

$$\underset{p_B^n}{Max} \pi_B^n = p_B^n \left( \theta_1 - \frac{1}{2} - \frac{p_B^n - p_A^o + d_A}{2t} \right) \text{ s.t. } d = \frac{p_A^o}{2}$$

From the FOC we obtain:

$$p_n = \frac{1}{4}p_A^o - \frac{1}{2}t + t\theta_1.$$

In the first stage of period 2 firm A solves the following problem:

$$\underset{p_A^o}{Max} \{p_A^o x_A^c + (p_A^o - d_A)(x_A - x_A^c)\}$$

from which we obtain:

$$p_A^o = \frac{2}{3}t + \frac{2}{3}p_B^n. \quad (18)$$

Thus,

$$p_A^o = \frac{2}{5}t(2\theta_1 + 1); \quad (19)$$

$$d_A = \frac{1}{5}t(2\theta_1 + 1); \quad (20)$$

$$p_B^n = \frac{2}{5}t(3\theta_1 - 1) \text{ as long as } \theta_1 > \frac{1}{3} \quad (21)$$

Note that if  $\theta_1 \leq \frac{1}{3}$ ,  $p_B^n = 0$  and so the best response of firm A is to quote  $p_A^o = t(1 - 2\theta_1)$ .

In the group of firm B's past consumers there is group of consumers who might be induced to switch given  $p_A^o$  and  $p_B^n$ . Under Losing Provider Led this consumers will contact firm B as a way to switch to A. Given this contact firm B offers a discount  $d$  as a way to retain these customers. The indifferent consumer between buying again from B at price  $p_B^o - d$  and switching to A is located at  $x_B$ :

$$p_A^n + t\theta_B = p_B^o + t(1 - \theta_B) - d$$

from which we obtain

$$\theta_B = \frac{1}{2} + \frac{p_B^o - p_A^n - d_B}{2t}.$$

Note that the indifferent consumer between contacting firm B is located at  $\theta_B^c$  such that:

$$\begin{aligned} U(p_B^o, d = 0) &\geq U(p_A^n) \\ p_B^o + t(1 - \theta_B^c) &= p_A^n + t\theta_B^c \\ \theta_B^c &= \frac{1}{2} + \frac{p_B^o - p_A^n}{2t}. \end{aligned}$$

Thus in the second stage firm B solves the following problem

$$\text{Max } (p_B^o - d_B)(\theta_B^c - \theta_B)$$

From the FOC it follows that  $d_B = \frac{p_B^o}{2}$ .

In the first stage of period 2 firm A solves the following problem:

$$\text{Max}_{p_A^n} \pi_A^n = p_A^n (\theta_B - \theta_1) \text{ s.t. } d_B = \frac{p_B^o}{2}$$

From the FOC we have that

$$p_A^n = \frac{1}{2}t + \frac{1}{4}p_B^o - t\theta_1.$$

In the first stage of period 2 firm B solves the following problem:

$$\text{Max}_{p_B^o} \{p_B^o (1 - \theta_B^c) + (p_B^o - d_B)(\theta_B^c - \theta_B)\}$$

It follows that

$$\begin{aligned} p_B^o &= \frac{2}{5}t(3 - 2\theta_1) \\ p_B^o - d_B &= \frac{1}{5}t(3 - 2\theta_1) \\ p_A^n &= \frac{2}{5}t(2 - 3\theta_1) \text{ for } \theta_1 < \frac{2}{3}. \end{aligned}$$

If  $\theta_1 \geq \frac{2}{3}$  it follows that  $p_A^n = 0$  and so the best response of firm B is to charge  $p_B^o = t(2\theta_1 - 1)$ . ■

**Proof of Corollary 5.** From the second-period equilibrium prices it is easy to obtain that second-period market shares are at the interior solution where  $\frac{1}{3} \leq \theta_1 \leq \frac{2}{3}$  given by

$$\begin{aligned} s_A^2 &= \theta_A + (\theta_B - \theta_1) = \frac{3}{5} - \frac{1}{5}\theta_1 \\ s_B^2 &= 1 - s_A^2 = \frac{2 + \theta_1}{5}. \end{aligned}$$

In this case it follows that  $s_A^2 \geq \frac{1}{2}$  ( $s_B^2 \leq \frac{1}{2}$ ) iff  $\theta_1 \leq \frac{1}{2}$ .

When  $\theta_1 \in [0, \frac{1}{3}]$

$$\begin{aligned} s_A^2 &= \theta_B = \frac{2\theta_1 + 2}{5} \\ s_B^2 &= 1 - s_A^2 = \frac{3 - 2\theta_1}{5}. \end{aligned}$$

Thus,

$$\begin{aligned} s_A^2 &\leq \frac{1}{2} \left( s_B^2 \geq \frac{1}{2} \right) \text{ iff } \theta_1 \in \left[ 0, \frac{1}{4} \right], \\ s_A^2 &\geq \frac{1}{2} \left( s_B^2 \leq \frac{1}{2} \right) \text{ iff } \theta_1 \in \left[ \frac{1}{4}, \frac{1}{3} \right]. \end{aligned}$$

Finally when  $\theta_1 \in [\frac{2}{3}, 1]$  :

$$\begin{aligned} s_A^2 &= \theta_A = \frac{2\theta_1 + 1}{5} \\ s_B^2 &= 1 - s_A^2 = \frac{2(2 - \theta_1)}{5}. \end{aligned}$$

Therefore,

$$\begin{aligned} s_A^2 &\geq \frac{1}{2} \left( s_B^2 \leq \frac{1}{2} \right) \text{ iff } \theta_1 \in \left[ \frac{3}{4}, 1 \right] \\ s_A^2 &\leq \frac{1}{2} \left( s_B^2 \geq \frac{1}{2} \right) \text{ iff } \theta_1 \in \left[ \frac{2}{3}, \frac{3}{4} \right]. \end{aligned}$$

■

**Proof of Proposition 4.** Consider the equilibrium choices of  $p_A^1$  and  $p_B^1$ . Firm A's and B's overall objective function are respectively

$$p_A^1 \theta_1 (p_A^1, p_B^1) + \frac{\delta t}{50} \left( 48 (\theta_1 (p_A^1, p_B^1))^2 - 36\theta_1 (p_A^1, p_B^1) + 19 \right)$$

and

$$p_B^1 (1 - \theta_1 (p_A^1, p_B^1)) + \frac{\delta t}{50} \left( 48 (\theta_1 (p_A^1, p_B^1))^2 - 60 (\theta_1 (p_A^1, p_B^1)) + 31 \right).$$

Thus from the FOC with respect to  $p_A^1$  and  $p_B^1$  we obtain firms A and B best-response functions respectively given by

$$p_A^1 (p_B^1) = \frac{125t + 125p_B^1 + 20t\delta - 95p_B^1\delta - t\delta^2}{250 - 70\delta}, \quad (22)$$

and

$$p_B^1(p_A^1) = \frac{125t + 125p_A^1 + 20t\delta - 95p_A^1\delta - t\delta^2}{250 - 70\delta} \quad (23)$$

from which we obtain

$$p_A^1 = p_B^1 = t \left( 1 - \frac{\delta}{25} \right).$$

Second-order condition for this problem is given by  $\frac{7\delta-25}{t(\delta+5)^2}$  which is negative for all  $\delta \in [0, 1]$ . ■

**Proof of Proposition 5.** The first-period equilibrium outcome is efficient under uniform pricing and under BBPD with and without retention strategies. Specifically, overall welfare in period 1 is equal to  $v - \frac{t}{4}$  in the three pricing regimes. In the second-period welfare with no discrimination,  $w_{nd}^2$  is

$$w_{nd}^2 = v - \int_0^{\frac{1}{2}} txdx - \int_{\frac{1}{2}}^1 t(1-x)dx = v - \frac{t}{4}.$$

With BBPD and retention discounts, the second-period welfare is  $w^2$

$$\begin{aligned} w^2 &= v - \int_0^{\theta_A} txdx - \int_{\theta_A}^{\theta_1} t(1-x)dx - \int_{\theta_1}^{\theta_B} txdx - \int_{\theta_B}^1 t(1-x)dx \\ &= v - \frac{27t}{100}. \end{aligned}$$

When firms engage in BBPD but retention offers are not permitted, the second period welfare is  $w^{2,NR}$  given by

$$\begin{aligned} w^{2,NR} &= v - \int_0^{\frac{1}{3}\theta_1 + \frac{1}{6}} txdx - \int_{\frac{1}{3}\theta_1 + \frac{1}{6}}^{\theta_1} t(1-x)dx - \int_{\theta_1}^{\frac{5}{6} - \frac{1}{3}\theta_1} txdx - \int_{\frac{5}{6} - \frac{1}{3}\theta_1}^1 t(1-x)dx \\ &= v - \frac{11t}{36}. \end{aligned}$$

Overall welfare in both periods, given by  $W = w^1 + \delta w^2$ , with no discrimination is given by

$$W^{nd} = v(1 + \delta) - 0.25t - 0.25t\delta$$

while under BBPD with retention discounts it equals

$$W^R = v(1 + \delta) - 0.25t - 0.27t\delta,$$

and, with BBPD and no retention offers it equals

$$W^{NR} = v(1 + \delta) - 0.25t - 0.30556t\delta.$$

Industry profits with no discrimination are equal to  $\pi_{ind}^{nd} = t + t\delta$ . From part (iv) of Proposition 4 it follows that industry profit under BBPD with retention activity is

$$\pi_{ind}^R = \frac{t(12\delta + 25)}{25}$$

while with no retention offers it is equal to

$$\pi_{ind}^{NR} = \frac{t(9 + 8\delta)}{9}$$

Overall consumer surplus in each of the three pricing regimes is respectively given by

$$CS^{nd} = v(1 + \delta) - 1.25t - 1.25t\delta$$

$$CS^R = v(1 + \delta) - 1.25t - 0.75t\delta$$

$$CS^{NR} = v(1 + \delta) - 1.25t - 1.1945t\delta$$

Thus, for any  $\delta > 0$ , it is straightforward to obtain (i), (ii) and (iii) of Proposition 5.

■

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