A GROWTH MODEL FOR THE QUADRUPLE HELIX

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Abstract. We develop a R&D-based growth model with productive public expenditure in order to frame the Quadruple Helix (QH) innovation concept, based on four helices: Academia & Technological Infrastructures, Firms, Government and Civil Society. Our motivation stems from acknowledgment that the relationship among these four helices and their joint impact on growth is in need of a theoretical framework. We aim to emphasise the importance to economic growth of innovation systems structured on these four helices. The introduced model confirms theoretically the notion that increases in: (i) complementarities between distinct productive units, or (ii) in productive government expenditure, lead to higher growth.

Keywords: economic growth, quadruple helix, innovation systems, government expenditure.

JEL Classification: O10, O18, O31.

1. Introduction

Wishing to contribute to the growing literature on innovation economies, we develop a R&D-based growth model with productive public expenditure in order to provide the Quadruple Helix (QH) innovation concept with a theoretical framework.

Today's economies are experiencing the emergence of a new nature of innovation, which distinguishes itself from that in the industrial era (OECD 2009), in which innovation consisted of technological developments performed by experts and research institutions in an environment characterised by a "silence is golden" culture.

Nowadays, innovation consists of all activities that create value by providing new solutions to concrete problems. Innovation arises as a result of co-creation between firms, citizens, universities and government, in a context marked by the existence of partnerships, collaborative networks and symbiotic relationships. The QH model describes this new economic environment.
The QH is a development of the Triple Helix (TH) innovation theory, according to which the establishment of creative links between three helices – Academia, Government and Industry – originates new knowledge, technology or products and services that are conveyed in fulfilment of society needs (e.g., Etzkowitz, Leydesdorff 2000; Etzkowitz, Klofsten 2005). Arguing that the TH is not sufficient for long-term innovative growth, and wishing to emphasise the importance of integrating the perspective of media-based and culture-based citizens, the QH adds a fourth helix to the innovation system – Civil Society (e.g., Lijemak 2004; Khan, Al-Ansari 2005). As Barroso (2010) also writes, modern economies’ growth requires cooperation between all economic agents, including social partners and Civil Society. Eriksson et al. (2005) also argue that in user-oriented innovation, users (Civil Society) are co-producers of innovation, their role being as important as those of research institutions, government support organisations and companies.

According to the QH theory, a country’s economic structure lies then on four helices – Academia & Technological Infrastructures, Firms, Government and Civil Society –, with economic growth being generated through continuous innovations.

Wishing to frame theoretically the equally important role of all the QH helices in economic growth, we develop a model that connects the four pillars and investigate analytically their interactions and joint impact on growth.

Assuming a one-sector-structure, our proposed QH model captures the notion that the whole society is involved in innovation, which occurs as a result of co-creation between the four helices, connected through networks, partnerships and symbiotic relationships.

Innovations are materialised by specialised productive units – Academia & Technological Infrastructures and Firms – that interact with and complement each other, within a cooperative, knowledge-sharing culture (e.g., Carayannis, Campbell 2006, 2009; Arnkil et al. 2010; McGregor et al. 2010). Technological Infrastructures consist in R&D infrastructures. They create networks, partnerships and associations to undertake R&D, and supply technical products and services (e.g., Etzkowitz, Leydesdorff 2000). As argued by Powell and Grodal (2005), Technological Infrastructures are also crucial in the codification of tacit knowledge in the form of finished inputs, hence enabling the transfer of knowledge through networks. Governments provide the financial support and the regulation system to promote the creation of links between Academia and Firms (science parks, business incubators and other bridge-institutions). Civil Society takes part in the economy by producing, contributing to innovation and demanding higher quality, forever innovative goods and services.

In the new innovation era, competing solely on pure technology has become harder. No single innovative agent has the resources or the competences to act alone. Interdependence of institutions is the result of the emerging innovation economies (OCDE 2009). Firms still maximise their profits, but business culture is changing from “silence is golden” into “we share”.

The concept of complementarities (see, e.g., Matsuyama 1995) seems adequate to capture this new innovation era in which all benefit from interaction, cooperation and
knowledge-sharing. Hence, following Thompson (2008), we assume the existence of complementarities between all entities that contribute in an intermediate level to final-good production — *Academia & Technological Infrastructures* and *Firms* — which we name the Intermediate Productive Units (IPUs).

Additionally, we capture the costly nature of investment in innovation, by assuming, also as in Thompson (2008), that there are internal costs to investment in both manufacture and innovation.

The *Government’s* role in the introduced model consists in undertaking productive public expenditure on education, health, infrastructures, technological and innovation services and regulations, which increases the productivity of all inputs. We use a Barro’s (1990) government expenditure specification.

*Civil Society* is engaged in production and innovation and also has a demand role, specified on the consumption side of our economy, where citizens (*Civil Society*) wish to consume innovative goods and services, all aggregated in the form of one final good.

The introduced model carries a second contribution to growth literature in the sense that it is a R&D-based growth model with public productive expenditure, which, according to Irmeng and Kuehnel (2009), is new to the literature on public expenditure and economic growth.

The remainder of the paper is organised as follows. Section 2 describes the model and its main results. Section 3 closes up the paper with some Conclusions.

2. Specification and results of the model

Innovation systems constitute environments in which public and private organizations and institutions — governments, universities, research centres, business communities, and funding/financing organizations — collaborate with and compete between each other, generating innovation through interaction of knowledge and information, human resources, financial capital and institutions (Carayannis, Campbell 2006, 2009). The participating elements in the QH innovation concept are, then, *Academia & Technological Infrastructures* (university laboratories and industrial R&D facilities), *Firms*, *Government* and *Civil Society*.

Innovation processes are not easy to define or manage. According to the Oslo Manual (OECD 2005), the strict definition of innovation is difficult to attain due to the complexity of innovation processes and the different ways in which they can occur according to types of firms and industries. Generally, *Academia* plays an important role as a source of knowledge and technology. However, the university-industry relationships can be difficult for firms to manage. For instance, new fields of knowledge with high rates of technological progress, like Nano-Bio-TIC, offer promising commercial opportunities, but pose considerable interaction problems between the different entities involved.

As Yawson (2009) writes, before the 2000’s the national system of innovation was formed by: (i) a set of institutions, which jointly or individually contributed to the development and diffusion of new technologies; and (ii) the *Government* which imple-
mented policies to influence the innovation process. In the 2000's, however, new con-
cepts regarding innovation systems have emerged, such as innovation systems, global
networking in value added and innovation, customers and users, systemic thinking and
sustainable innovation.

West and Farr (1989: 16), for instance, define innovation as the "... intentional intro-
duction and application within a role, group or organization of ideas, processes, products
or procedures, (...) designed to significantly benefit role performance of the group,
the organization or the wider society". For Johnson (1992), innovation is a continuous
cumulative process involving not only radical and incremental innovation but also the
diffusion, absorption and use of innovation. For the OECD (2009), innovation consists
in creating value by developing new solutions to specific problems.

We aim to frame this wide definition scope for innovation while also emphasising the
idea that the new nature of innovation is essential for smart, inclusive and sustainable
economic growth (Europe’s 2020 Strategy). Hence the introduced model carries the
assumption that the whole society takes part in the innovation process, i.e. we specify
a one-sector structure in which innovation is undertaken with the same technology as
that of manufacture, by the whole population.

Innovations are materialised in intermediate goods and services (inputs). The final good
(aggregate output) is produced using Labour (Civil Society), public expenditure and all
the existing inputs. Each input’s physical units are produced by Firms and Academia &
Technological Infrastructures.

The model needs to be understood in a circular perspective: All the existing interme-
diate goods and services are used to produce aggregate output. In turn, aggregate output
can be either consumed or invested. Investment consists of innovation expenditure plus
physical capital accumulation and is required to innovate and produce more intermediate
goods and services, so that the economy grows.

2.1. Production side – Technology Equation

The single final good (aggregate output) Y(t) is produced with constant labour (all the
economy’s citizens, i.e., Civil Society) L(t); public expenditure G(t); and the inputs
(intermediate goods and services) x_i(t), produced by a number A(t) of intermediate pro-
ductive units i, (i = 0 ... A). Each intermediate productive unit is associated with one
innovation i. Innovations arise as a result of co-creation between Academia & Techno-
logical Infrastructures, Government, Firms and Civil Society, in a one-sector structure
framework.

2.1.1. Government expenditure

The Government's role in this economy (our innovation system) consists in providing
a pure public good – in the form of government expenditure on education, health, in-
frastructures, technological and innovation services and regulations, which increases
the productivity of all productive factors in the same way. That is, we follow Barro
(1990) and assume that productive government expenditure is a flow variable. Thus, in
Equation 1, the flow of productive government expenditure $G$ is a constant fraction $\tau$ of output $Y$ for all $t$:

$$G(t) = \tau Y(t), \quad 0 < \tau < 1. \quad (1)$$

The government's budget is balanced in all periods. Assuming, for simplicity, zero-public-debt and zero-consumption-taxes, the government's budget constraint is:

$$G(t) = T(t) = \tau Y(t). \quad (2)$$

In Equation 2, $T(t)$ are taxes, that is, total government revenue, at time $t$.

2.1.2. Intermediate productive units (IPUs)

We assume that Academy & Technological Infrastructures and Firms have an identical productive role in this economy. They constitute the intermediate productive units (IPUs) $i$, ($i = 0 \ldots A$), and produce the (physical) inputs $x_i(t)$.

With the goal of capturing the "benefic-for-all" interactions and cooperation between the existing IPUs in innovation systems (e.g., Carayannis 2006, 2009), we assume that IPUs are complementary to each other in the production of aggregate output. Matsuyama (1995), for instance, regards complementarities as a relevant feature of industrialised economies, essential in explaining economic growth, business cycles and underdevelopment.

As in Thompson (2008), building on Evans et al. (1998), we specify that the inputs of the IPUs enter complementarily in the production function for $Y(t)$.

2.1.3. Final good

The production function for $Y(t)$ is $Y(t) = L(t)^{1-\alpha-\beta} G(t)^{\beta} \left( \int_0^{A(t)} x_i(t)^\gamma dt \right)^{\phi}$, which, substituting $G(t)$ by its equivalent according to Equation 1, becomes:

$$Y(t) = \tau^{1-\beta} L(t)^{1-\beta} \left( \int_0^{A(t)} x_i(t)^\gamma dt \right)^{\phi/(1-\beta)}, \quad \gamma \phi = \alpha, \quad \frac{\phi}{1-\beta} > 1. \quad (3)$$

In Equation 3, the parameter restriction $\gamma \phi = \alpha$ is imposed to preserve homogeneity of degree one, and assumption $\frac{\phi}{1-\beta} > 1$ is made so that the IPUs inputs $x_i$ are complementary to one another; i.e., so that an increase in the quantity of one input increases the marginal productivity of the other inputs.

Assuming that it takes one unit of physical capital $K(t)$ to produce one physical unit of any type of IPUs input, $K(t)$ is related to inputs $x_i(t)$ by the rule:

$$K(t) = \int_0^{A(t)} x_i(t) dt. \quad (4)$$

2.1.4. Innovation

An innovation consists in any project useful for concrete problem solving, leading to the production of a technological or non-technological manufactured good or service. We wish to capture the idea that the whole society is involved in the innovation process.
Florida (2002), for instance, writes that creativity comes from all kinds of people who are the critical resources of modern economies. Karnitis (2006), for example, goes further, highlighting that all social classes must work together in order to achieve common goals, with social inclusion being a prerequisite for growth and development.

Participation of the whole society in the innovation process is possible due to the development of new information and communication technologies (Ginevicius, Korsakiene 2005), allowing individuals to be more active in society.

Following Rivera-Batiz and Romer (1991), we assume the one-sector structure in that innovation is undertaken with the same technology as that of the final good and IPUs inputs. We further assume that innovation i involves a cost equal to \( P_i \hat{z} \) units of foregone output, where \( P_i \) is the fixed cost of one new innovation in units of foregone output, and \( \hat{z} \) represents an additional cost of innovation i in terms of foregone output, meaning a higher innovation cost for higher indexed innovations. Like in Evans et al. (1998), this extra cost is introduced in order to avoid explosive growth.

Accommodating Anagnostopoulos (2008)'s argument, innovation expenditure is specified as part of total capital investment expenses. With zero depreciation for simplicity, total investment in each period \( \dot{W}(t) \) is equal to physical capital accumulation \( \dot{K}(t) \) plus innovation expenditure \( P_i \dot{A}(t) \hat{z} \):\[ \dot{W}(t) = \dot{K}(t) + P_i \dot{A}(t) \hat{z}. \] (5)

Bearing in mind Equation 5, it follows that total capital \( W(t) \) is equal to physical capital plus innovation capital:
\[ W(t) = K(t) + P_i A(t) \frac{\hat{z}+1}{\xi+1}. \] (6)

It will be later shown that, in a Balanced Growth Path (BGP), \( Y \) and \( W \) in Equations 3 and 6, respectively, grow at the same rate, which means that we can write aggregate output as a linear function of total capital:
\[ Y(t) = BW(t). \] (7)

In Equation 7, \( B \) is the marginal productivity of total capital, which is constant in a BGP.

2.1.5. Costly investment

Agreeing with Benavie et al. (1996) and Romer (1996), our model contemplates investment costs. Following Thompson (2008), we assume that investment in total capital \( W(t) \) involves an internal cost, that is, installing \( I(t) = \dot{W}(t) \) new units of total capital requires spending an amount given by:
\[ J(t) = I(t) + \frac{1}{2} \theta \frac{I(t)^2}{W(t)}. \] (8)

In Equation 8, \( C(I(t),W(t)) = \frac{1}{2} \theta \frac{I(t)^2}{W(t)} \) represents the Hayashi's (1982) installation cost, with \( \theta > 0 \) standing for the adjustment cost parameter.
Closing up this one-sector-framework, the economy’s budget constraint is given by Equation 9:

\[ I(t) + \frac{1}{2} \theta \frac{I(t)}{W(t)} = Y(t) - G(t) - C(t). \]  

(9)

The equilibrium investment rate maximises the present discounted value of cash flows. The current-value Hamiltonian is:

\[ H(t) = BW(t) - I(t) - \frac{1}{2} \theta \frac{I(t)^2}{W(t)} + q(t)I(t). \]  

(10)

In Equation 10, \( q(t) \) is the market value of capital and the transversality condition of this optimization problem is \( \lim_{t \to \infty} e^{-rt} q(t)W(t) = 0 \), with \( r \) representing the real interest rate. We solve the model for a particular solution, the BGP, for which growth rates are constant. We will suppress the time argument from now onwards, whenever that causes no confusion. Having in mind that the growth rate of output is \( g_Y = g_W = g = \frac{I}{W} \), the first-order condition, \( \frac{\partial H}{\partial I} = 0 \), is equivalent to:

\[ q = 1 + \theta g. \]  

(11)

Equation 11 says that, in a BGP solution, \( q \) is constant.

The co-state equation, \( \frac{\partial H}{\partial W} = rq - \dot{q} \), is equivalent to:

\[ \dot{q} = rq - \left( B + \frac{1}{2} \theta g^2 \right), \]

which, in a BGP solution, becomes:

\[ q = \frac{B + \frac{1}{2} \theta g^2}{r}. \]  

(12)

Equation 12 also implies a constant interest rate \( r \).

Let us now build the Technology Equation. Final good producers are price takers in the market for inputs. In equilibrium they equate the rental rate on each input with its marginal productivity. The demand curve faced by each IPU is given by Equation 13:

\[ \frac{\partial Y(t)}{\partial x_j(t)} = R_j(t) = \frac{\alpha}{\theta} \frac{\beta}{1 - \beta} \frac{1}{\tau^{1-\beta} L(t)^{1-\beta}} x_j(t)^{\gamma - 1} \left[ \int_0^{A(t)} x_i(t)^{\gamma - 1} \right]^{1 - \beta}. \]

(13)

Turning now to the IPUs’ production decisions: Once invented, the physical production of each unit of the input requires one unit of capital. In each period, the monopolistic IPU maximises its profits, taking as given the demand curve for its good:

\[ \max_{x_j(t)} \pi_j(t) = R_j(t)x_j(t) - rqx_j(t), \]

which leads to the mark-up rule in Equation 14:

\[ R_j = \frac{rq}{\gamma}. \]  

(14)

At time \( t \), in order to enter the market and produce the \( A \)th input, an IPU must spend up-front an innovation cost given by \( P_A A(t)^{\delta} \), where, as mentioned earlier, \( P_A \) is the fixed
cost of one new innovation, in units of foregone output, and $\xi$ represents an additional cost of innovation $i$ in terms of foregone output. Hence, the dynamic IPU’s zero-profit condition $P_A A(t)^\xi = \int_0^\infty e^{-r(s-t)} \pi_j(s) ds$ is, assuming no bubbles, equivalent to:

$$\xi G_A = r - \frac{\pi_j}{P_A A^{\xi}}. \quad (15)$$

The model’s symmetry implies that $R_f(t) = R(t)$, $x_f(t) = x(t)$ and $\pi_f(t) = \pi(t)$. Hence $R(t)$ is rewritten as:

$$R = \Omega_R A^{\frac{1-\beta}{1-\beta}} x^{1-\beta}. \quad (16)$$

In Equation 16, $\Omega_R = \frac{\alpha}{1-\beta} \tau^{1-\beta} L^{1-\beta}$ is a constant. Then, profits $\pi(t) = (1-\gamma)R(t)x(t)$ are given by Equation 17:

$$\pi = \Omega_\pi A^{\frac{1-\beta}{1-\beta}} x^{1-\beta}, \quad (17)$$

with $\Omega_\pi = (1-\gamma)\Omega_R$. And $x$ is equal to:

$$x = A^{\xi} \left( \frac{\Omega_R}{R} \right)^{\frac{1-\beta}{1-\beta}}. \quad (18)$$

In Equation 18, we impose the parameter restriction $\xi = \frac{\phi - (1-\beta)}{(1-\beta) - \alpha}$, so that we can obtain a BGP solution (see Evans et al. 1998).

In a balanced growth path, the interest rate and the shadow-value of capital are constant and hence so is $R$. It then follows, from Equation 16, that we must have

$$\left( \frac{\phi - 1 + \beta}{1-\beta} \right) g_A = \left( \frac{\alpha - 1 + \beta}{1-\beta} \right) g_x,$$

that is: $g_x = \xi g_A$, $\xi = \frac{\phi - (1-\beta)}{(1-\beta) - \alpha}$.

Symmetry also implies that Equation 4 simplifies to $K = Ax$, meaning that $g_K = (1+\xi)g_A$. Likewise, the production function can now be written as Equation 19:

$$Y = \frac{\beta}{\tau^{1-\beta} L^{1-\beta}} A^{1-\beta} x^{1-\beta}, \quad (19)$$

whose time-differentiation gives $g_Y = \left( \frac{\phi + \alpha \xi}{1-\beta} \right) g_A = (1+\xi)g_A$, allowing us to change Equation 15 into:

$$g_Y = \frac{1+\xi}{\xi} \left( r - \frac{\Omega_Y}{\frac{\alpha}{R^{(1-\beta) - \alpha}}} \right), \quad \Omega_Y = \frac{(1-\gamma) \Omega_R^{1-\beta}}{\xi}. \quad (20)$$

Equation 20 is our Technology Equation. It unites the equilibrium BGP pairs of interest rate and economic growth rate $(r, g)$ on the production side of this economy.

2.2. Consumption side – the Euler Equation

Civil Society is composed, in this model, by all citizens of the economy, assumed to be infinitely lived, homogeneous, well informed and cultivated. Civil Society wishes
to consume innovative goods and services, all aggregated in the form of final good \( Y \) whose production requires innovation.

Analytically, we can simply adopt the standard specification for intertemporal consumption, as it enables us to convey our interpretation of Civil Society's demand role. Hence, citizens solve an intertemporal optimization problem, that is, they maximise the discounted value of their representative utility (Equation 21), subject to a budget constraint (Equation 22):

\[
\max_{C(t)} \int_0^\infty e^{-\rho t} \frac{C(t)^{1-\sigma}}{1-\sigma} \, dt, \\
\text{s.t.} \quad \dot{E}(t) = rE(t) + w(t) - C(t) - T(t),
\]

where variable \( C \) is consumption of \( Y \) in period \( t \), \( \rho \) is the rate of time preference, and \( \frac{1}{\sigma} \) is the elasticity of substitution between consumption at two periods in time. Variable \( E \) stands for total assets, \( r \) is the interest rate, \( w \) is the wage rate, and it is assumed that each inhabitant provides one unit of labour per unit of time. The transversality condition is \( \lim_{t \to \infty} \mu(t)E(t) = 0 \), where \( \mu(t) \) is the shadow price of assets.

The resulting Civil Society's consumption decisions (in terms of long run consumption growth) are given by the familiar Euler Equation 23:

\[
g_c = \frac{\dot{C}}{C} = \frac{1}{\sigma}(r - \rho).
\]

2.3. General equilibrium

2.3.1. Analytical solution

Time-differentiation of the investment Equation 5, \( \frac{\dot{W}}{W} = \frac{\dot{K}}{K} + \frac{\dot{A}}{A} \frac{A^{1+\xi}}{W} P_A \), tells us that \( W \) grows at the same rate as \( Y \), that is \( g_W = (1 + \xi)g_A \).

Then, the economy's budget constraint Equation 9 tells us that a constant growth rate of \( W \) implies that consumption grows at the same rate as output. In fact, \( \dot{W} = Y - G - C - \frac{1}{2} \frac{I(t)^2}{W(t)} \), is equivalent to:

\[
g_W = \frac{Y}{W} - \frac{G}{W} - \frac{C}{W} - \frac{1}{2} \theta g^2.
\]

According to Equation 24, a constant \( g_W \) requires that \( \left( \frac{\dot{Y}}{W} \right) = \left( \frac{\dot{G}}{W} \right) + \left( \frac{\dot{C}}{W} \right) \). As \( G \) and \( W \) grow at the same rate as \( Y \), then \( C \) must also grow at the same rate as \( Y \).

Summing up, with labour constant, the per-capita economic growth rate is given by \( g_C = g_Y = g_K = g_W = g = (1 + \xi)g_A \).

Hence, the general equilibrium solution is obtained by solving the system of the two Equations 20 and 23, in two unknowns, \( r \) and \( g \). Recalling Equation 11, the system to be solved is:
\[
\left\{
\begin{aligned}
g &= \frac{1}{\sigma} (r - \rho) \\
g &= \frac{1 + \xi}{\xi} \left[ r - \frac{\Omega}{\frac{\alpha}{\left( r + r \rho \right) \left( 1 - \beta \right) - \alpha}} \right], \quad r > g > 0.
\end{aligned}
\right.
\]  

(25)

In Equation 25, \( \Omega = \gamma \frac{(1 - \gamma)}{(1 - \beta) - \alpha} \frac{1 - \beta}{\Omega R^{(1 - \beta) - \alpha}} \), and \( \Omega R = \frac{\alpha}{1 - \beta} \frac{\beta}{\tau^{1 - \beta} L^{1 - \beta}} \).

We impose restriction \( r > g > 0 \) so that (i) present values will be finite; and (ii) our solution(s) have positive interest and growth rates.

The Euler Equation 23 is linear and positively sloped in the space \((r, g)\). The Technology Equation 20 is nonlinear, as shown in the Appendix. The model delivers, however, a unique solution.

**Proposition 1.** The QH innovation model has a unique solution for \( \sigma > 1 \) and \( \Omega \frac{1 - \alpha - \beta}{1 - \beta} > \rho \).

**Proof.** Defining two new variables and rewriting our system, we can show that the proposed model has a unique solution. Our new variables are:

\[ Y = \theta g; \quad Z = r(1 + \theta g), \]

which allows us to rewrite the system as:

\[
\left\{
\begin{aligned}
Z &= \frac{\sigma}{\theta} (Y + 1)(Y + \eta) \\
Z^\omega &= \frac{\lambda}{Y + \mu}
\end{aligned}
\right.
\]  

(26)

In Equation 26, \( \omega = \frac{\alpha}{(1 - \beta) - \alpha}, \quad \lambda = \frac{\theta \Omega}{\sigma - \frac{\xi}{1 + \xi}}, \quad \mu = \frac{\rho \theta}{\sigma - \frac{\xi}{1 + \xi}}, \quad \eta = \frac{\rho \theta}{\sigma}. \)

Our restrictions become \( Y > 0, Z \geq \frac{1}{\theta} Y(Y + 1) \).

To ensure that \( r > g \), we impose \( \sigma > 1 \) so that the Euler Equation 23 lies above the 45° line. This implies that \( \lambda, \mu \) and \( \eta \) are all positive. Hence, the first equation of the rewritten system defines a strictly decreasing curve \( Y \mapsto Z(Y) \) from \( Z(0) = \left( \frac{\Omega}{\rho} \right)^{\omega} \) to \( Z(\infty) = 0 \), while the second equation defines a strictly increasing curve \( Y \mapsto Z(Y) \) from \( Z(0) = \rho \) to \( Z(\infty) = \infty \). Thus, the system has a unique solution in the region \( Y > 0 \) iff \( \Omega > \rho^{\omega + 1} \) (which is equivalent to \( \Omega \frac{1 - \alpha - \beta}{1 - \beta} > \rho \)). The second restriction is also met because \( Z = \frac{\sigma}{\theta} (Y + 1)(Y + \eta) > \frac{1}{\theta} Y(Y + 1) \).
2.3.2. Numerical solutions

Given the nonlinearity of the Technology Equation, we resort to solving the system through a numerical exercise. For the numerical determination of our unique general equilibrium solution, the invariant parameter values considered are:

\[ \sigma = 2; \quad \rho = 0.02; \quad \alpha = 0.4; \quad \beta = 0.3; \quad \gamma = 0.1; \quad \phi = 4; \quad \xi = 11; \quad L = 1; \quad \tau = 0.15, \]

where the values for \( \alpha, \gamma \) and consequently \( \phi = \frac{\alpha}{\gamma} \) are the same as those used by Evans et al. (1998) in their numerical example. Consequently \( \xi = \frac{\phi - (1 - \beta)}{(1 - \beta) - \alpha} = 11. \) The values for the preference parameters \( \sigma \) and \( \rho \) are in agreement with those found in empirical studies such as Barro and Sala-i-Martin (1995). The value for parameter \( \tau \) is in agreement with Irmen and Kuehnel (2009). Population is often chosen to have unity value, so as not to give relevance to the scale-effects prediction that growth depends on the size of the economy, present in many growth models.

We then obtain several possible general equilibrium solutions for different values of parameters \( \theta \) and \( P_A \). The chosen values for \( \theta \) and \( P_A \) are in line with Whited (1992) and Connolly and Valderrama (2005), respectively (Table 1).

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<td>( g = 0.0383 )</td>
<td>( g = 0.0177 )</td>
<td>( g = 0.0116 )</td>
</tr>
<tr>
<td></td>
<td>( r = 0.0787 )</td>
<td>( r = 0.0374 )</td>
<td>( r = 0.0252 )</td>
</tr>
</tbody>
</table>

For expositional purposes, selecting the combination \( \theta = 1.5 \) and \( P_A = 6 \), the system in Equation 25 is:

\[
\begin{align*}
g & = 0.5r - 0.01 \\
g & = \left( \frac{12}{11} \right) \left[ r - \frac{0.000283}{(r + 1.5rg)^{\frac{4}{3}}} \right]
\end{align*}
\]

Figure 1, with \( r \) on the horizontal axis and \( g \) on the vertical axis, shows the BGP general equilibrium of this economy for the chosen parameters values. We can add that higher values of \( \theta \) and \( P_A \) (for instance, \( \theta = 50 \) and \( P_A = 100 \)) do not alter significantly the configuration of the model.
2.3.3. Additional results

**Corollary 1.** *Everything else constant, an increase in the public investment parameter, \( \tau \), leads to an increase in the equilibrium growth rate.*

**Proof.** Looking at the rewritten model (26):

\[
\begin{align*}
Z &= \frac{\sigma}{\theta} (Y + 1)(Y + \eta) \quad (A) \\
Z^\omega &= \frac{\lambda}{Y + \mu} \quad (B),
\end{align*}
\]

denoting our curves (A) and (B), curve (A) is positively sloped and curve (B) is negatively sloped in the space \((Z, Y)\). An increase in \( \tau \) implies an increase in \( \Omega \), meaning that curve (B) shifts to the right. The new equilibrium has higher values for \( Z \) and \( Y \), as illustrated in Figure 2. Given that \( g = \frac{Y}{0} \), this implies a higher value for the growth rate \((dr = 0)\).

**Corollary 2.** *Everything else constant, an increase in the complementarities parameter, \( \phi \frac{\rho}{1 - \beta} \), leads to an increase in the equilibrium growth rate.*
Proof. As in Corollary 1, an increase in \( \frac{\phi}{1-\beta} \) implies an increase in \( \alpha \), hence an increase in \( \Omega \), meaning that curve (B) shifts to the right. The new equilibrium has higher values for \( Z \) and \( Y \), as illustrated in Figure 2. Given that \( g = \frac{Y}{\theta} \), this implies a higher value for the growth rate \( (dr = 0) \).

3. Conclusions

We have developed a R&D-based growth model with productive public expenditure in order to provide the QH innovation concept with a first analytical theoretical framework. Within the introduced model, we analyse questions concerning productive public expenditure, the importance to economic growth of complementarities between the different productive units in innovation economies, the relevance of considering the costly nature of investment, and policies to achieve higher economic growth.

As Carayannis and Campbell (2009) refer, QH encompasses structures and processes of the gloCal Knowledge Economy and Society. Innovation systems generate a democracy of knowledge, whose creation is transdisciplinary, non-linear, hybrid and shared. Yawson (2009), for example, writes that advances in biotechnology, ICT and nanotechnology have stimulated innovation and convergence, but at the same time, have revealed the importance of adequate regulations, and have introduced a need for society awareness. Civil Society has thus become an essential helix of innovation systems. The developed QH model considers the innovation economy with four helices: Academia & Technological Infrastructures (university laboratories and industrial R&D facilities), Firms, Government and Civil Society, all equally important for smart, inclusive and sustainable economic growth.

The emerging new nature of innovation carries the implication that no single innovative agent has the resources or the competences to act alone. Interdependence of institutions is, indeed, the distinguishing feature of innovation economies. Specifying the beneficial interactions and cooperation between productive units through the presence of complementarities between all the intermediate productive units, the introduced model conveys analytically the result that an increase in complementarities in the innovation economy does increase economic growth.

Yawson (2009) also argues that the QH innovation theory can give orientation in regard of economic policy. Recognizing that innovation by creative citizens determines the success of a country’s innovation strategy, innovation systems start with a national innovation goal, which is interpreted through the four helices’ perspectives in an integrated form. In the QH innovation model here proposed, Government provides a pure public good, in the form of productive expenditure on education, health, infrastructure, technological and innovation services and regulations, which increases the productivity of all inputs. The model illustrates analytically that an increase in productive public expenditure does increase the economic growth rate of QH economies.
Having framed analytically the new nature of innovation and its impact on economic growth, the next step is to capture this economic dynamics empirically. As Godin (2011) discusses, to measure a country’s innovation performance and its impact on the country’s economic performance constitutes a true challenge.

References


APPENDIX

In order to analyse the shape of the Technology Equation (20), and as it is impossible to isolate \( r \) on one side of the equation, we rewrite it as \( F(r, g) = 0 \) and apply the implicit function theorem, so as to obtain, in the neighbourhood of an interior point of the function, the derivative \( \frac{dr}{dg} \):

\[
F(r, g) = \xi g - \left(1 + \xi \right) r + \left(1 + \xi \right) \Omega_Y r^{1-\beta-\alpha} \left(1 + \theta g\right)^{\frac{-\alpha}{1-\beta-\alpha}} = 0,
\]

which leads to:

\[
\frac{dr}{dg} = \frac{\frac{dF(r, g)}{dg}}{\frac{dF(r, g)}{dr}} = \frac{\xi - \left(\frac{\alpha}{1-\beta-\alpha}\right) \theta \left(1 + \xi \right) \Omega_Y r^{1-\beta-\alpha} \left(1 + \theta g\right)^{\frac{-\alpha}{1-\beta-\alpha}}}{\left(1 + \xi \right) + \left(\frac{\alpha}{1-\beta-\alpha}\right) \left(1 + \xi \right) \Omega_Y r^{1-\alpha} \left(1 + \theta g\right)^{\frac{\beta-1}{1-\beta-\alpha}}}.
\]

Hence, our nonlinear Technology Equation is positively sloped when:

\[
\frac{\xi}{\left(1 + \theta g\right)^{\beta-1} \left(1 + \xi \right) \Omega_Y} < \frac{\alpha}{1-\beta-\alpha},
\]

and negatively sloped otherwise.


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