

## 1. INTRODUCTION

Precast prestressed concrete (PC) structures have several advantages compared to cast-in-place structures; they can be constructed more rapidly and are generally more robust and durable. They are made of individual elements and assembled with various types of connections. For beams, these connections may require severe reductions of the cross-section at the ends, called dapped-ends. The abrupt change of cross-section in a beam results in a complex flow of internal stresses, which are typically highly concentrated at the re-entrant corner. Such regions in an element are called disturbed regions (D-regions) [1, 2]. According to the PCI Design Handbook [2] dapped-end beams may fail in any of the five modes schematically represented in Fig. 1: (1) Flexure (cantilever bending) and axial tension in the extended end; (2) Direct shear at the junction between the dapped and undapped zone of the member; (3) Diagonal tension failure at the re-entrant corner; (4) Diagonal tension failure in the extended end; and (5) Diagonal tension failure in the undapped zone. Numerous researchers have analyzed and experimentally investigated these failure modes. Using the strut and tie method, Reynold [3], Mattock and Chan [4], Mattock and Theryo [5] and Hwang and Lee [6] have all proposed equations for predicting these failure modes and presented design criteria for dapped-end beams. Chen [7] tested four dapped-ends with identical geometry and reinforcement ratio, but different reinforcement layouts. The results showed that the reinforcement arrangement influences the capacity of the elements, and that the provisions given in the ACI 318-08 code [8] are conservative. Lu et al. [9] theoretically and experimentally investigated the shear resistance of 12 dapped-end beams, again finding that the PCI Design Handbook [2] provisions are conservative, and suggested new design proposals.

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The load carrying capacity (hereafter capacity, for convenience) of dapped-end beams may be insufficient for reasons such as design errors, code changes, increases in loads, or structural damage. One option to increase the capacity of the dapped-end regions is to use fiber-reinforced polymers (FRP) using the externally bonded reinforcement (EBR) technique. FRPs are viable solutions for strengthening or retrofitting reinforced concrete (RC) elements, and several guidelines for strengthening RC structures with FRPs have been published recently [10-12]. However, these guidelines do not refer specifically to FRP strengthening of dapped-end beams, partly because the variations in geometry, material and loading conditions at their dapped ends hinder the establishment of clear criteria for robustly classified strengthening configurations. In a series of tests, Huang and Nanni [13] verified that FRPs can increase the capacity of dapped-end beams with “mild steel and no mild reinforcement” [13], and proposed a method for strengthening dapped-end beams with FRPs that was found to be “satisfactory and conservative” [13]. They too showed that dapped-end reinforcement designed according to the PCI Design Handbook [2] is very conservative. Gold et al. [14] strengthened dapped-end beams of a three-story parking garage that were deficient in shear resistance with FRP. Due to the lack of design provisions at that time, they carried out a series of tests to verify the effectiveness of the FRP strengthening and predictive performance of their design approach. The FRP strengthening systems doubled the resistance of the beams, confirming their effectiveness. Tan [15] experimentally investigated the efficiency of several FRP configurations for strengthening dapped-end beams with deficient shear resistance, varying in both fiber types and mechanical anchorage systems for the FRP. The results showed that glass fiber reinforced polymers (GFRP) provided greater improvements in terms of ultimate load than carbon FRP plates and carbon fiber sheets, and the tested mechanical anchorage devices enhanced exploitation of the FRP systems’ strengthening capacity by preventing their debonding. The empirically based strut and tie model they

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derived was applied to predict increases in the shear capacity of the dapped-end beams, and proved to be sufficiently accurate for the type of beams tested. More recently, in a large series of experiments Taher [16] assessed the effectiveness of the following techniques for improving the capacity of dapped-end beams: externally bonding steel angles; anchoring unbonded steel bolts in inclined, pre-drilled holes; externally applying steel plate jackets; and wrapping carbon fiber around the beam stem. Tests with 50 small-scale rectangular beams indicated that the FRPs were the most viable solution for strengthening/retrofitting applications. Using the strut and tie analogy, Taher also derived a regression model to estimate the capacity of the FRP-strengthened dapped-end beams, which reportedly provided “reasonable” predictions [16], but he did not consider any possible scale effects of the beams tested for deriving the model.

The current study presents an experimental program prompted by a real case application.

To the authors’ knowledge, only four experimental investigations on dapped-end beams strengthened with FRPs have been reported [13-16]. The specimens tested in the previous four experimental programs and the research presented here are shown (at the same scale) in Fig. 2. Clearly, there are major differences in the specimens used in these research programs, in terms of the configuration of the dapped-end, position of the loading, shear spans, and size of the beams (see Table 1). Therefore, the work presented in this paper enriches the experimental database on FRP-strengthened dapped-end beams and provides both experimental and numerical assessments of the behavior of large, FRP-strengthened (and unstrengthened reference) dapped-end beams.

## **2. THE CONTEXT OF THIS WORK**

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A single-storey industrial hall was constructed using 20 m long identical precast/prestressed beams, with 1800x660 mm cross-section, serving as supports for the roof purlins (Fig. 1). Immediately after assembly of the structure eight beams had diagonal cracks corresponding to the third mode of failure, see Fig. 1. The inclination of the crack angle varied between  $40^\circ$  to  $50^\circ$  with respect to the longitudinal axis of the beam. Initially the dapped-ends were designed using the strut and tie method according to the Romanian codes [17] and EC2 [18] for a reaction force of 800 kN positioned 400 mm from the re-entrant corner. An inspection revealed that the dapped-ends were not in full contact with the supporting columns. Consequently, the position of the reaction force was displaced by an additional 275 mm, resulting in the diagonal cracking. Hence, the dapped-end beams were re-assessed, considering the new lever arm (675 mm). At this stage, the reaction force in the beam was about 500 kN. The widely used strut and tie models proposed by Schlaich et al. [19] and Martin [20] (Fig. 3a,b), indicated that the dapped-end beams' capacity was 600 and 590 kN, respectively, based on the prescribed safety factors for the materials and the loads. Hence, they had a deficit in capacity of ca. 200 kN, and a strengthening solution using EBR carbon fiber reinforced polymer (CFRP) plates was proposed to meet it. The International Federation for Structural Concrete's guideline [10], recommends a strain limitation of 4‰ for FRP plates. However, due to the variations in FRP strengthening applications, there are serious doubts about the suitability of this limit; thus a further aim of the presented research was to assess its validity under the examined conditions.

### **3. RESEARCH PROGRAM**

#### **3.1. Experimental investigations**

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Two beams, each with two dapped-ends, were cast. Since the investigations focused on the dapped-ends, the total length of the element was reduced from 20 m to 6.24 m by removing the flexural span of the beam; the tensile prestressed reinforcement was replaced with an equivalent steel reinforcement area; and the height of the cross-section of the beam was reduced from 1.8 m to 1.5 m. These modifications should have had no significant influence on the failure mode, as observed at the job site (mode 3, in Fig. 1.). The arrangement, spacing, diameter and strength class of the reinforcements were identical to those of the original beams (Fig. 4).

### **3.1.1. Material tests**

The material properties were determined from laboratory tests, in conformity with [21] for concrete and [22] for steel. A concrete with a maximum aggregate size of 16 mm and mean compressive cube strength of  $56 \text{ N/mm}^2$  was used, corresponding to a C45/55 concrete strength class. Characteristic values of the tensile properties of the steel reinforcements are presented in Table 2.

### **3.1.2. Test setup, instrumentation and test procedure**

The dapped-end specimens were tested according to the schematic test configuration shown in Fig. 5, by applying a monotonic force in increments of 50 kN by a hydraulic jack. After each step the loading was maintained and the cracks were mapped.

Before casting, one strain gauge was applied in the horizontal bar closest to the re-entrant corner (S1), and another was glued in the vertical bar nearest the re-entrant corner (S2), Fig. 4. The strains recorded by the gauges are classified as illustrated by the following example:

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S1-RC2-T is the strain registered in steel reinforcement (S) in vertical (1) direction for the retrofitted (R) concrete element 2 (C2) during the experimental test (T).

The horizontal displacements of the specimens were monitored using four linear variable differential transducers (LVDTs): one at the top (M7) and one at the bottom (M4) of the nib; plus one at the top (M5) and one at the bottom of the undapped portion (M6). Three LVDTs were also used to measure the vertical displacements: one placed in the front face at the bottom of the nib (M1), one on the edge of the dapped-end at the re-entrant corner (M2), and one on the bottom edge of the undapped portion (M3), Fig. 6. The “displacement” term in the presented load-displacement diagrams is defined as the difference in positions recorded by the LVDTs at points M1 and M3.

In the experimental program, the four dapped-end specimens were tested in the following sequence. First, the C1 element was tested to failure, to obtain reference measurements of the capacity of the *in situ* dapped-end beams before strengthening. Then the remaining three dapped-ends (C2, C3 and C4) were tested up to 800 kN; the design load of the original dapped-ends. Finally, the C2, C3 and C4 elements were repaired, strengthened as described below and retested to failure.

### **3.1.3. Strengthening of the elements**

The pre-cracked elements were strengthened by using two systems of EBR CFRPs, in three different solutions. The aims of the strengthening solutions were to increase the capacity up to the design load and to delay the yielding initiation of the steel reinforcement. After the initial tests of the C1-C4 elements the surface of the concrete was ground, the dust and impurities were removed with compressed air, the strengthening systems were applied and cured during seven days. New strain gauges (labeled G in Figs. 10c, 14c and 17c) were

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installed to monitor the strains developed in the FRPs. An identical test setup was used for both the unstrengthened (C) and retrofitted (RC) specimens. Table 3 presents mechanical and geometrical properties of the FRPs. For the field application, strengthening Systems 1 and 2 were used. Since the  $0^\circ/90^\circ$  layout (RC4) provided the longest anchorage length this configuration was preferred in an attempt to avoid premature debonding. However, in some cases the purlins obstructed application of this layout (RC4, Fig. 17), hence the  $45^\circ/90^\circ$  layout (RC2, Fig. 10) was adopted. For comparison, strengthening System 3 was also applied in the experimental program. The amount of carbon fibers (CFRP) used in System 3 was modified to provide equivalent strength to that of Systems 1 and 2.

## **3.2 Numerical investigation**

### **3.2.1. Modeling strategy**

The numerical analysis presented in this paper was carried out *a posteriori* to the tests. Due to malfunction of some of the installed gauges, it was not possible to register the corresponding strains during the tests. Therefore, to better understand the failure progress of the tested elements in general, and the behavior of the FRPs in particular, the tests were simulated in 2D using ATENA software. The aim of this analysis was to obtain information regarding the development of strains in the steel reinforcement after strengthening and in the FRP up to failure.

The numerical modeling was carried out in two steps. First, several full geometry finite element models were constructed and analyzed using characteristic values reported by the manufacturers for the concrete, steel and FRP reinforcement in order to calibrate the experimental test setup. In the second step the specimens were modeled using average values

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of the material properties, determined from standardized tests. In the latter phase the boundary conditions were also calibrated, to account for elastic deformations in the upper supports of the reaction frame of the test setup. Here, only the results from the second modeling phase are presented. Due to software limitations it was not possible to simulate the phased process of the strengthening intervention (as the CFRP systems were applied when the specimens had already cracked). Therefore, in the numerical modeling the CFRP systems were considered to have been applied to uncracked specimens. This limitation has certain drawbacks for modeling the strain development in the CFRP systems. However, as will be shown, its consequences are of minor significance in the context of the field application. In a similar manner as for the laboratory tests, the displacement plotted in all the diagrams was computed as the difference between the M1 and M3 measuring points presented in Fig. 6.

The standard incremental and iterative Newton-Raphson method for material nonlinear structural analysis was used in the numerical simulations, based on the finite element method (FEM). The specimens were modeled with a mesh of 8-node serendipity plane stress finite elements. A Gaussian integration scheme with 2x2 integration points was used for all the concrete elements. The steel bars, CFRP laminates and CFRP sheets were modeled with 2-noded perfectly bonded embedded truss elements (one degree of freedom per node) [23].

The strains monitored during the analysis are labeled similarly to those from the experimental tests, but the character N is added to distinguish the experimental from the numerical results. For instance, the designation S1-RC2-N refers to strains obtained numerically at the position coinciding with the S1 strain gauge, in the numerical simulation of the specimen strengthened with the RC2 configuration.

### **3.2.2. Properties of the materials in the simulations**

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### **3.2.2.1. Concrete**

The constitutive model for concrete used in the analysis is a fracture-plastic model that combines constitutive sub-models for tensile and compressive behavior, as presented in the ATENA user manual [23]. This fracture model employs the Rankine failure criterion and exponential softening, with the hardening/softening plasticity component based on the Menétrey-William failure surface [23]. The concrete post-cracking tensile behavior was simulated by the softening function illustrated in Fig. 7a in combination with the crack band theory [23].

### **3.2.2.2. Steel bars and carbon fiber reinforced polymer systems**

The reinforcement was modeled using discrete bars, whose tensile and compression behavior were simulated using the stress-strain relationship illustrated in Fig. 7b. After the peak strength,  $f_u$ , the stress was reduced to 10% of  $f_u$  so that internal stress redistribution could be assured in the numerical computations. Table 2 presents values used for characterizing the stress-strain relationships of the different types of bars composing the reinforcement.

FRP materials exhibit linear-elastic behavior until failure. In a similar manner as for the steel bars, a descending branch was also modeled, so that after the peak strength was reached (Fig. 7c), the FRP material could not support further loads. The FRP material was introduced using several discrete lines (modeled as bars perfectly bonded to the substrate), each equivalent to a 50 mm wide strip. The same procedure was used for the plates, except that the strips were 25 mm wide. The values of the mechanical properties of the CFRP systems are indicated in Table 3. The predictive performance of the simulations, described in Section 4, show that this simplified approach is acceptable for the purposes of the presented numerical analyses.

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### **3.2.3. Boundary conditions**

To allow eventual separation of the bottom surface of the specimen in contact with the supporting RC floor, contact elements without tensile capacity were adopted. The elastic deformation of the test supporting system was calibrated based on the experimental results obtained for the reference specimen, and then integrated in all the numerical simulations.

## **4. RESULTS OF THE EXPERIMENTAL PROGRAM**

The research focused on the strengthening system used in a practical application. Results are presented for two loading scenarios: the first for loadings up to 800 kN, for which the dapped-ends were designed, while the second simulates the ultimate limit state (ULS) loading condition of each strengthened element. Results from both experimental and numerical investigations are reported and discussed in terms of: load-displacement behavior, strain distributions in the steel and FRP reinforcements, the increase in capacity of the dapped-ends provided by FRP strengthening, crack patterns and failure modes. Strains in the extremities of the FRPs were not monitored during the tests, but estimates of their values were obtained from the numerical analysis. The numerical analysis accurately predicted the load-displacement responses of all specimens up to the failure load recorded in the tests. After this load, the models for elements RC2 and RC4 overestimated the FRP contribution, because the peeling-off mechanism could not be simulated in the 2D analysis. Up to the yielding strain of the horizontal steel rebars, the strains in the steel and FRPs computed from numerical analysis agreed well with values recorded in the tests. Between the yielding load and failure load, the strains in the steel bars determined by FEM analysis show some differences from

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those recorded in the tests. These differences may be attributed to the tension stiffening effect present in the beams but not implemented in the numerical models. Furthermore, the strains recorded experimentally only represent the strains in the zones where the strain gauges (SG) were installed, while the FEM analysis provided average strains for the steel bars.

#### **4.1. Test results for the C1 specimen**

The C1 (control) element was tested to failure. Up to 700 kN just a single crack formed, starting from the re-entrant corner at an angle of  $60^\circ$  with respect to the longitudinal axis of the beam. As the load increased, several additional cracks formed, and finally cracks were uniformly distributed around the re-entrant corner, as shown in Fig. 8a. The test was stopped at 1600 kN, when a displacement of 30 mm was registered. This relatively high deformation was due to yielding of the tensile reinforcement and local crushing of the compressed concrete under the supports. At that moment the major measured crack was 3.5 mm wide, see Fig. 8b. The strain gauges attached to the reinforcements did not function during this test. However, the FEM analysis (Fig. 9) indicates that yielding of the vertical (S2-C1-N) and horizontal (S1-C1-N) steel reinforcements occurred at loads of about 865 and 970 kN, respectively. At the design limit, i.e. 800 kN, the strains were 1.75 ‰ in horizontal (S1) and 2.20 ‰ in vertical (S2) reinforcements, as subsequently confirmed in the tests of C2, C3 and C4 specimens. The FEM analysis indicated that failure occurred at 1573 kN (Fig. 9a) by crushing of the concrete in compression and yielding of the tensile reinforcement as indicated in Fig. 9b.

#### **4.2. Test results for the C2 and RC2 specimens**

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Specimen C2 was tested up to 800 kN. The first crack (1), occurred at 600 kN and started from the re-entrant corner at 42° angle with respect to the longitudinal axis of the beam (Fig. 10). Between 600 and 650 kN three other cracks (2, 3, 4) formed near the first crack. From 650 kN until the target load two additional cracks (5, 6) formed (Fig. 10a). The strain gauge attached to the horizontal steel reinforcement (S1-C2-T) recorded 1.75 ‰ strain at 800 kN, close to the yielding strain (2.2 ‰). The strain gauge installed on the vertical reinforcement only recorded values up to 200 kN, therefore these recordings will not be commented upon. After the test the specimen was unloaded, retrofitted with FRP, and retested (Fig. 10b).

Compared with the reference specimen (C1), this retrofitted element (RC2) had a higher capacity and stiffness, especially above the deflection corresponding to a loading of 800 kN (Fig. 12a), due to the FRP systems delaying development of the cracks in the concrete and decreasing the strain in the reinforcements (Fig. 12b).

The stress-strain relationship of the RC2 element was almost linear up to 1300 kN. Up to this load the existing cracks did not develop further, but new three cracks (7, 8, 9) formed in the nib (Fig. 10b). This shows the capacity of the applied FRP systems to limit damage that occurred in the C2 test phase, promoting stress redistribution. Between 1300 and 1350 kN a new crack (10) formed around the inclined FRP plates and extended towards the bottom part of the vertical plates (Figs. 10 and 11). The element started to fail at 1670 kN, by the inclined plates on the right face of the strengthened dapped-end successively peeling-off (Fig. 11). At this load the strain in the FRP was 5.5 ‰ (G5-RC2-T, see Fig. 10c); 32 % of the FRP's ultimate strain. The failure ended in a brittle manner at 1760 kN, when the inclined FRP plate on the left face of the dapped-end peeled-off, and caused the vertical FRP to peel off (Fig. 11). At failure the strains measured in the inclined (G4-RC2-T) and vertical (G3-RC2-T) FRP plates were 7.2 and 4.9 ‰, respectively (Fig. 12c). The strain measured in the steel

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reinforcement (S1-RC2-T in Fig. 4) at a load of 800 kN was 1.20 ‰; 31 % less than the strain at this load in the reference specimen (S1-C2-T).

The strain distribution along the FRP, determined from the numerical simulations, for a load of 1620 kN, just before the peeling-off process started, is shown in Fig. 13. For this load the maximum strain recorded at the bottom-end of the inclined plates was 4.1 ‰, while at the bottom-end of the vertical plates only marginal strains were recorded. This confirms observations from the experiments that the inclined plates initially debonded and subsequently the vertical plates.

The FEM analyses also indicated that if the FRPs had been mechanically anchored, the capacity of the dapped-ends could have been increased to 1917 kN. The maximum strain in the inclined FRP (17 ‰) was reached at 1845 kN, just before peak load. After the first strip reached its ultimate strength, the remaining FRP bands failed consecutively up to the point at which the peak load was reached. At peak load a strain of 9.50 ‰ in the vertical FRP was obtained. This suggests that a debonding failure could have been avoided if the inclined FRP plates had been mechanically anchored. If so, the numerical analysis indicates that the failure mode could have been changed from FRP peeling-off to yielding of the horizontal reinforcement together with failure of the inclined FRP and crushing of the compressed concrete.

### **4.3. Test results for the C3 and RC3 specimens**

Specimen C3 was tested up to 800 kN. The first crack (labeled 1 in Fig. 14), started at 400 kN and developed at a 45° angle from the re-entrant corner. Between 400 to 600 kN three other cracks (2, 3, 4) formed near the first crack, followed by two additional cracks (5, 6) by the time the target load was reached (Fig. 14a). At 800 kN the strains recorded by the gauges

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attached to the horizontal steel reinforcement (S1-C3-T) and vertical steel rebar (S2-C3-T) were 1.81 and 1.76 ‰, respectively. Hence, the reinforcement was close to the yielding level (2.2 ‰). At this loading the specimen was unloaded, retrofitted according to the FRP configuration illustrated in Fig. 14b, and retested.

The stress-strain relationship of the RC3 specimen was almost linear up to 900 kN but above 450 kN the existing cracks (1, 4, 6) reopened, and at each 50 kN load increment the cracks gradually widened, and the fibers of the sheets started rupturing progressively, with an accompanying typical loud noise. The strain gauges bonded to the composite (Fig. 14c) became damaged at a load of about 500 kN. However, at 800 kN the strain gauges installed on horizontal (S1-RC3-T) and vertical (S2-RC3-T) steel reinforcements indicated strains of 1.20 and 1.50 ‰, respectively. Thus, the FRP system reduced strains in these reinforcements, relative to those in the reference specimen, by 33% and 15%, respectively. The RC3 specimen had marginally higher ultimate capacity and higher stiffness up to 900 kN than the C1 element, due to the delayed cracking.

The numerical analysis accurately predicted the behavior of the RC3 specimen up to the maximum load recorded in the test (1587 kN vs 1618 kN), see Fig. 15a. The strains in the horizontal and vertical steel rebars obtained by the FEM analysis were in good agreement with the strains recorded in the tests up to 1100 kN, at which the rebars started yielding (Fig. 15b). The strains recorded by the G3-RC3-T and G4-RC4-T gauges (Fig. 14c) correlated well with the strains predicted by the numerical simulations while these strain gauges were working properly, see Fig. 15c. However, the deviation of the strains predicted at the position of the G5-RC4-T gauge from the measured values were relatively high, possibly because of its impairment after it was crossed by the principal crack during early loading stages. The strain distribution along the FRP (Fig. 16) clearly indicates its failure mode; fiber rupture of

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all the sheets along the plane of the principal crack, with values tending to zero at sheet-ends, and local crushing of concrete in the most compressed zone of the dapped-end.

#### **4.4. Test results for the C4 and RC4 specimens**

Specimen C4 was also tested up to 800 kN. The first crack (labeled 1 in Fig. 17), occurred at 600 kN and started from the re-entrant corner at  $46^\circ$  angle with respect to the longitudinal axis of the beam. At 650 kN two other cracks (2, 3) formed near the first crack, followed by four additional cracks (4, 5, 6, 7) by the time target load was reached (Fig. 17a). The strain gauge attached to the horizontal steel reinforcement (S1-C4) recorded 1.44 ‰ strain at 800 kN. Due to malfunctioning of the gauge attached to the vertical reinforcement its strains were only measured up to 500 kN. At the target load the specimen was unloaded, retrofitted according to the FRP configuration represented in Fig. 17b, and retested.

The stress-strain relationship of the RC2 element was almost linear up to 1300 kN. Up to this load the existing cracks did not develop further and no other new cracks formed. Between 980 and 1000 kN a new crack (8) started around the horizontal plates, extending towards the vertical plates. In the next load steps several new cracks (9, 10, 11) appeared, distributed concentrically with respect to the re-entrant corner.

The specimen started to fail at 1670 kN, by the horizontal plates successively peeling-off from both faces of the strengthened dapped-end (Fig. 18). The maximum measured strains in the FRP at this load were 4.12 ‰ on the left face (G4-RC4-T) and 6.65 ‰ on the right face (G5-RC4-T), see Figs. 17c and 19c; 24 % and 39 %, respectively, of the FRP's ultimate strain. The failure ended suddenly, at 1680 kN, when the vertical plates on both faces failed by debonding, preceded by the horizontal plates peeling-off (Fig. 18). At this load the strain measured in the vertical FRP plate was 6.72 ‰ (G3-RC2-T). The strain measured in the steel

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reinforcement (S1-RC4-T) at 800 kN was 1.20 ‰, 15% lower than the strain recorded in the reference specimen (S1-C4-T) at this load.

Compared with the reference specimen (C1), the retrofitted element RC4 showed a higher capacity and stiffness, especially above the deflection corresponding to a load of 800 kN (Fig. 19a). Its overall behavior was similar to that of the RC2 specimen.

As shown in Fig. 19a, the numerical analysis accurately predicted its load-displacement behavior up to the failure load recorded in the experimental test. Similarly to the results for the RC2 element, the model overestimated the final resistance of the specimen, for reasons already mentioned. Up to the yielding limit of the horizontal rebars, the strains in the steel and FRP reinforcements computed by the FEM agreed well with those recorded in the experimental tests. After yielding started and up to the failure load the strains in the steel determined by the FEM started diverging from (but remained similar to) those recorded in the tests, see Fig. 19c.

Fig. 20 shows the strain distribution along the FRPs obtained from the numerical analysis at failure load. At 1680 kN, just before the first peeling-off recorded in the test, a maximum strain of 2.01 ‰ was recorded at the ends of the horizontal strips, while the strains near the ends of the vertical strips were about 1 ‰.

Similarly to the findings for RC2, the FEM analysis also indicated that if the FRPs had been mechanically anchored (to prevent peeling-off failure), the capacity of the dapped-end could have been increased up to 1844 kN. The maximum strain in the horizontal FRP (17 ‰) was reached at 1830 kN, just before peak load. After the failure of the first strip, the other horizontal strips failed sequentially up to the attainment of the peak load. The vertical FRP developed a maximum strain of 11.12 ‰ at the peak load.

## 5. DISCUSSION AND CONCLUSIONS

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This paper describes experimental and nonlinear numerical analyses of four dapped-end beams strengthened with different FRP configurations. This research program was designed to identify the most suitable FRP strengthening configuration among those used in a real field application. The beams to be strengthened on-site were designed to resist a load of 800 kN. Due to an error during the building phase, the lever arm of the support system increased, and diagonal cracking appeared, starting from the reentrant corner of the dapped-ends of these beams. Using the design methods for dapped-ends described in the PCI handbook [2], in the Romanian and European standards [17, 18] resulted in conservative values for the capacity of these elements. The cited methods predicted a load of about 600 kN for the yielding initiation of the steel reinforcement, considering the prescribed safety factors for both the materials and the loads. However, the test with the C1 dapped-end reference specimen indicated a yielding load of 865 kN, and in tests with the other (C2-C4) dapped-end specimens yielding did not occur up to 800 kN. There were two reasons for these differences between the design and empirically determined values. The first was the presence of real material in the tests, with significantly higher strength than the design values, especially for the reinforcement. The second was the influence of the shear span and the element cross-section dimensions. The procedure applied to generate the abovementioned standards does not seem to have been optimized for the design of large dapped-end beams, such as those investigated in this work, hence the conservative nature of the predictions. Therefore, further research is needed to improve the existing design methods for large dapped-end beams.

After subjection to the target load (800 kN), three specimens were retrofitted with different CFRP strengthening configurations, and then retested. The strengthening configurations applied in the RC2 and RC4 specimens were those applied to real beams on-site. Compared to the control element C1, the strengthened dapped-ends exhibited a slight increase in

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ultimate load capacity. For elements RC2 and RC4 the capacity increases (4 % and 10%, respectively) were limited by the failure mode, i.e. debonding of the FRP. The numerical analysis showed that if mechanical anchorages had been used to avoid this premature failure the capacity could have been increased by up to 20%. However, at a loading of 800 kN the strengthening configurations applied to the RC2 and RC4 specimens reduced the stress in the reinforcement by 31% and 16 %, respectively, and increased their stiffness, resulting in capacity gains of approximately 18% at the first yielding in the reinforcement, with respect to the reference specimen C1.

The strengthening system applied in the RC3 specimen was ineffective in terms of increasing the capacity and stiffness due to the relatively small ultimate strain of the CFRP composites used in this configuration. The gradient of strains in the CFRP section crossed by the main cracks caused local failure of fibers in early stages of loading during the crack opening process. It is believed that higher capacity could be obtained if larger content of FRPs were applied. However, above a certain content of FRP, the failure mode might change from tensile rupture of the fibers to debonding; if so the capacity could only be increased if anchorage devices were used to avoid premature debonding.

Despite the simplifications applied in the 2D simulation, the results from the numerical analysis matched the test measurements fairly well. These results helped elucidation of the behavior of the tested specimens and assessment of the strain distribution in the reinforcements. During the tests, it was difficult to see with the naked eye when and where the debonding of the FRP plates started. However, the strain distribution on the FRP plates showed that the horizontal and inclined plates are most prone to debond, starting from the inner end of the dapped-ends due to the limited bond length. The numerical analyses also showed that the vertical plates had adequate anchorage length, and that the debonding of the

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inclined/horizontal plates triggered the debonding of the vertical plates, in accordance with the test observations.

Tests showed that systems 1 and 2 are viable strengthening solutions, confirming that the retrofiting methods proposed for the field application are reliable for limiting the crack openings.

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## **Nomenclature**

- H Height of the experimental specimens
- B Width of the web cross-section
- h Height of the dapped-end
- $l_p$  Recess of the dapped-end

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- a Shear span according to PCI provisions
- $f_{ct}$  Tensile strength of concrete
- $G_f$  Mode I fracture energy of concrete
- wc Crack opening at the complete release of stress
- $f_y$  Yield strength of reinforcement
- $f_u$  Tensile strength of reinforcement
- $\varepsilon_y$  Strain in reinforcement at yielding
- $\varepsilon_u$  Strain in reinforcement at maximum force
- $\varepsilon_{ult}$  Strain in reinforcement at failure (conventional value, adopted only for finite element modeling convenience).