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Behaviour-Based Price Discrimination with Elastic Demand*

Rosa Branca Esteves† Carlo Reggiani‡

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Abstract

Behaviour-based price discrimination is typically analysed in a framework characterised by inelastic demand. This paper provides a first assessment of the role of elastic demand on the competitive effects of behaviour-based price discrimination. Our results show that if demand is elastic enough, behaviour-based price discrimination leads to demand expansion which has a positive effect on overall welfare.
JEL: D43, L13.
Keywords: behaviour-based price discrimination, elastic demand.

1 Introduction

Behaviour-based price discrimination (BBPD) is a very commonly adopted business practice: firms gather information on the past shopping behaviour of consumers and make use of it by proposing deals to new and old consumers. The increasing diffusion of internet as a marketplace and the unprecedented ability to store huge amount of data is enhancing firms' knowledge of consumers' preferences and, consequently, the predominance of this type of practices. Examples of firms using BBPD include supermarkets, web retailers, telecom companies, restaurants and many others.

One characteristic of BBPD in most of these examples is that consumers' decision does not only involve choosing a firm but also the amount of good(s)

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purchased. The existing literature on the topic, instead, has mainly focused on the case in which consumers' demand is inelastic: each consumer decides whether to consume or not the good supplied but not how many units of it. This paper relaxes this assumption and introduces elastic demand in the context of BBPD.

The economics literature on BBPD is relatively new but has already successfully explained a number of practices that the standard categories of price discrimination could not capture. The seminal paper of Fudenberg and Tirole (2000) has first shown that firms have an incentive to poach consumers although this is reducing their profits.

A common assumption adopted by this literature is that firms are competing in a unit demand framework *à la Hotelling*: the implication is that the role of demand elasticity on the competitive effects of BBPD has been mostly overlooked. The assumption may be justified by the challenge posed by elastic demand in a Hotelling framework. Nero (1999) and Rath and Zhao (2001) seem to be the first to tackle the issue. They use quadratic utility preferences to show that a location than price Hotelling game with elastic demand has a unique equilibrium. Both papers emphasise the role of the transport cost to reservation price ratio in determining the optimal location chosen by firms. Anderson and De Palma (2000) introduce constant elasticity of substitution (CES) utility in a spatial framework to analyse issues related to localized and global competition. Wenzel and Guo (2009, 2011) use the same system of preferences to address the optimality of firm entry in spatial models and the role of information and transparency on the variety supplied by the market, respectively.

In this paper we also adopt a CES utility function to introduce elastic demand in the analysis of competitive BBPD. These preferences allow us to provide a closed form solution to a two periods BBPD model with elastic demand. Moreover, our results confirm that the assumption of inelastic demand may not be so innocuous. In particular, price discrimination is beneficial to consumers' independently of the effect of BBPD on average prices; the demand expansion effect of BBPD always increases consumers' surplus. Unlike the inelastic case, in our context the demand expansion effect also implies that BBPD can be welfare enhancing: if demand is sufficiently elastic, the higher volume of transactions more than compensates for the increase in transport costs.

The rest of the paper is structured as follows. Section 2 introduces the model. Section 3 sets the benchmark case with no discrimination. Section 4 solves the model when firms practice behaviour based price discrimination. Section 5 discusses the implications of BBPD and Section 6 concludes.

2 The model

Two firms, $i = A, B$, produce at zero marginal cost[1] a nondurable good and compete over two periods, $t = 1, 2$. On the demand side, there is a large number of consumers whose mass is normalized to one. In each period a consumer can either decide to buy the good from firm A or from firm B, but not from both. We assume that the two firms are located at the extremes of a unit interval $[0, 1]$, and consumers are uniformly distributed along this interval. A consumer situated at $x \in [0, 1]$ is at a distance $d_A(x) = x$ from firm A and at distance $d_B(x) = 1 - x$ from firm B and τ is the unit transport cost. Transport cost is linear in distance and does not depend on the quantity purchased. Note that the location of a consumer x represents his relative preference for firm B over A while $\tau > 0$ measures how much a consumer dislikes buying a less preferred brand. A consumer's brand preference x remains fixed for both periods. Following Anderson and De Palma (2000) and Gu and Wenzel (2009, 2011), we write the utility of a consumer buying from firm i as:

$$U_i(x) = v - V(q_i) - \tau d_i(x) - p_i q_i,$$

in which v is the gross utility of consuming the good, $V(q_i)$ is the utility derived by consuming q_i units of the good, $td_i(x)$ is the total transport cost of buying from firm i and $p_i q_i$ is the consumer's expenditure. We shall assume throughout that the reservation value is high enough that all consumers purchase in both periods. Assuming preferences display constant elasticity of substitution, type-x consumers' net utility of buying q_A units from firm A at the marginal price p_A can be written as:

$$U_A = v - \frac{1-\sigma}{\sigma} q_A^{-\frac{\sigma}{1-\sigma}} - \tau x - p_A q_A = v - \frac{1}{\sigma} p_A^{\sigma} - \tau x,$$

where $q_i = p_i^{1-\sigma}$ and $\sigma \in (0, 1]$. Notice that this demand specification encompasses the standard Hotelling setup as $\sigma = 1$ and perfect competition as $\sigma \to 0$. Similarly, if consumer x buys q_B units form firm B, the net utility is: $v - \frac{1}{\sigma} p_B^{\sigma} - \tau(1 - x)$. These preferences imply that the consumer indifferent between buying from firm A or B is located at $x = \frac{1}{2} + \frac{p_B^{\sigma} - p_A^{\sigma}}{2\tau\sigma}$. Firms demand are given by $D_A = x p_A^{\sigma-1}$ and $D_B = (1 - x) p_B^{\sigma-1}$ while profits are $\pi_A = x p_A^{\sigma}$ and $\pi_B = (1 - x) p_B^{\sigma}$ respectively. In each period firms act simultaneously and non-cooperatively. In the first period, consumers are *anonymous* and firms quote the same price for all consumers. In the second period, whether or not a consumer bought from the firm in the initial period reveals that

[1] The assumption of zero marginal costs can be relaxed without altering the basic nature of the results derived throughout the model.

consumer's brand preference. Thus, as firms have the required information, they will set different prices to their own customers and to the rival's previous customers. If price discrimination is not adopted (for example, if it is forbidden) firms quote again a single price to all consumers. Firms and consumers discount future profits using a common discount factor $\delta \in [0, 1]$.

3 Benchmark: no discrimination

Suppose that for some reason (e.g. regulation, costs of changing prices...) firms in the second period can not price discriminate. In that case, the two-period model reduces to two replications of the static equilibrium. To solve for this equilibrium, consider the one period model, and let p_A and p_B denote the prices set by firms A and B, respectively. Firm A solves the following problem:

$$\max_{p_A} \pi_A = \left\{ p_A^\sigma \left(\frac{1}{2} + \frac{p_B^\sigma - p_A^\sigma}{2\tau\sigma} \right) \right\}.$$

From the first-order condition, the best response function is:

$$p_A = \left(\frac{\tau\sigma + p_B^\sigma}{2} \right)^{\frac{1}{\sigma}}.$$

Firm B's best response function, similarly, is:

$$p_B = \left(\frac{\tau\sigma + p_A^\sigma}{2} \right)^{\frac{1}{\sigma}}.$$

Solving for the equilibrium, the following proposition can be stated (without proof):

Proposition 1 *In the no discrimination benchmark case equilibrium prices in each period are equal to:*

$$p^{nd} = (\tau\sigma)^{\frac{1}{\sigma}},$$

and each consumer buys:

$$q^{nd} = (\tau\sigma)^{\frac{\sigma-1}{\sigma}}.$$

The industry equilibrium profits are:

$$\Pi^{nd} = (1 + \delta)\,\tau\sigma.$$

4 Behaviour-based price discrimination

Price discrimination is now feasible. In period 1 firms cannot recognise customers so they set a single first period price, denoted p_i^1. Consumers' first period choices reveal information about their brand preferences, so firms can set their second period prices accordingly: in the second period, each firm can offer two prices, one to its own past customers, denoted p_i^o, and another price to the rival's previous customers, denoted p_i^r. To derive the subgame perfect equilibrium, the game is solved using backward induction from the second period.

4.1 Second-period pricing

As in Fudenberg and Tirole (2000) the consumers first-period decisions will lead to a cut-off rule, so that first-period sales identify two intervals of consumers, corresponding to each firm's turf. Suppose that at given first-period prices p_A^1 and p_B^1, there is a cut-off x_1^* such that all consumers with $x < x_1^*$ bought from firm A in period 1. Thus, firm A's turf is the interval $[0, x_1^*]$, while firm B's turf is the remaining $[x_1^*, 1]$. On firm A's turf (i.e. firm A's strong market and firm B's weak market), firm A offers price p_{oA}, while firm B offers price p_{rB}. The marginal consumer, x_{2A} who is indifferent between buying again from firm A and switching to firm B is identified by the following condition:

$$\frac{1}{\sigma}p_{oA}^{\sigma} + \tau x_{2A} = \frac{1}{\sigma}p_{rB}^{\sigma} + \tau(1 - x_{2A}),$$

implying:

$$x_{2A} = \frac{1}{2} + \frac{p_{rB}^{\sigma} - p_{oA}^{\sigma}}{2\tau\sigma}.$$

Each consumer in the market segment $[0, x_2]$ buys $q_{oA} = p_{oA}^{\sigma-1}$ units from firm A in the second period and each consumer in the market segment $[x_2, x_1^*]$ switches to firm B in period 2 and buys $q_{rB} = p_{rB}^{\sigma-1}$ units. Thus, firm A's demand from retained customers in period 2 is given by $D_{oA} = x_{2A}p_{oA}^{\sigma-1}$ and, similarly, firm B's demand from switching customers is: $D_{rB} = (x_1^* - x_{2A})\, p_{rB}^{\sigma-1}$.

Firm A's second period profit from old customers is:

$$\pi_{oA} = p_{oA}D_{oA} = x_{2A}p_{oA}^{\sigma} = \left(\frac{1}{2} + \frac{p_{rB}^{\sigma} - p_{oA}^{\sigma}}{2\tau\sigma}\right) p_{oA}^{\sigma},$$

and firm B's second period profit from switching customers is

$$\pi_{rB} = p_{rB}D_{rB} = (x_1^* - x_{2A})\, p_{rB}^{\sigma} = \left(x_1 - \frac{1}{2} - \frac{p_{rB}^{\sigma} - p_{oA}^{\sigma}}{2\tau\sigma}\right) p_{rB}^{\sigma}.$$

On its turf, firm A chooses p_{oA} to maximise π_{oA} for any given p_{rB} leading to the following best response function:

$$p_{oA} = \left(\frac{\tau\sigma + p_{rB}^{\sigma}}{2} \right)^{\frac{1}{\sigma}},$$

Firm B's best response function on firm A's turf is instead:

$$p_{rB} = \left(\frac{\tau\sigma\left(2x_1 - 1\right) + p_{oA}^{\sigma}}{2} \right)^{\frac{1}{\sigma}}.$$

Solving for the equilibrium and using an analogous reasoning for firm B first period turf leads to the following proposition.

Proposition 2 *When firms can recognise their old and the rivals' customers and price discriminate second-period equilibrium prices and quantities are: (i) if* $\frac{1}{4} \leq x_1 \leq \frac{3}{4}$ *:*

$$\begin{aligned} p_{oA} &= \left[\frac{\tau\sigma\left(2x_1 + 1\right)}{3} \right]^{\frac{1}{\sigma}} \text{ and } p_{rA} = \left[\frac{\tau\sigma\left(3 - 4x_1\right)}{3} \right]^{\frac{1}{\sigma}}, \\ q_{oA} &= \left[\frac{\tau\sigma\left(2x_1 + 1\right)}{3} \right]^{\frac{\sigma-1}{\sigma}} \text{ and } q_{rA} = \left[\frac{\tau\sigma\left(3 - 4x_1\right)}{3} \right]^{\frac{\sigma-1}{\sigma}}, \end{aligned}$$

and:

$$\begin{aligned} p_{oB} &= \left[\frac{\tau\sigma\left(3 - 2x_1\right)}{3} \right]^{\frac{1}{\sigma}} \text{ and } p_{rB} = \left[\frac{\tau\sigma\left(4x_1 - 1\right)}{3} \right]^{\frac{1}{\sigma}}, \\ q_{oB} &= \left[\frac{\tau\sigma\left(3 - 2x_1\right)}{3} \right]^{\frac{\sigma-1}{\sigma}} \text{ and } q_{rB} = \left[\frac{\tau\sigma\left(4x_1 - 1\right)}{3} \right]^{\frac{\sigma-1}{\sigma}}. \end{aligned}$$

(ii) if $x_1 \leq \frac{1}{4}$ *:*

$$\begin{aligned} p_{oA} &= \left[\tau\sigma\left(1 - 2x_1\right)\right]^{\frac{1}{\sigma}} \text{ and } p_{rA} = \left[\frac{\tau\sigma\left(3 - 4x_1\right)}{3} \right]^{\frac{1}{\sigma}}, \\ q_{oA} &= \left[\tau\sigma\left(1 - 2x_1\right)\right]^{\frac{\sigma-1}{\sigma}} \text{ and } q_{rA} = \left[\frac{\tau\sigma\left(3 - 4x_1\right)}{3} \right]^{\frac{\sigma-1}{\sigma}}, \\ p_{oB} &= \left[\frac{\tau\sigma\left(3 - 2x_1\right)}{3} \right]^{\frac{1}{\sigma}} \text{ and } p_{rB} = 0, \\ q_{oB} &= \left[\frac{\tau\sigma\left(3 - 2x_1\right)}{3} \right]^{\frac{\sigma-1}{\sigma}} \text{ and } q_{rB} = 0. \end{aligned}$$

(iii) if $x_1 \geq \frac{3}{4}$:

$$\begin{aligned}
p_{oA} &= \left[\frac{\tau\sigma\left(2x_1+1\right)}{3}\right]^{\frac{1}{\sigma}} \quad \textit{and } p_{rA}=0, \\
q_{oA} &= \left[\frac{\tau\sigma\left(2x_1+1\right)}{3}\right]^{\frac{\sigma-1}{\sigma}} \quad \textit{and } q_{rA}=0, \\
p_{oB} &= \left[\tau\sigma(2x_1-1)\right]^{\frac{1}{\sigma}} \quad \textit{and} \quad p_{rB}=\left[\frac{\tau\sigma\left(4x_1-1\right)}{3}\right]^{\frac{1}{\sigma}}, \\
q_{oB} &= \left[\tau\sigma(2x_1-1)\right]^{\frac{\sigma-1}{\sigma}} \quad \textit{and} \quad q_{rB}=\left[\frac{\tau\sigma\left(4x_1-1\right)}{3}\right]^{\frac{\sigma-1}{\sigma}}.
\end{aligned}$$

Proof See Appendix.

4.2 First-period pricing

Consider now the equilibrium first-period pricing and consumption decisions. If firms have no commitment power, their market shares in the first period will affect their second period pricing and profits. Thus, forward looking firms take this interdependence into account when setting their first period prices. As consumers are not myopic they anticipate the firms' second period behaviour. Suppose the first-period prices lead to a cut-off x_1 that is in the interior of the interval $[0,1]$ then the marginal consumer must be indifferent between buying q_{1A} units in the first period at price p_{1A}, and buying q_{rB} units next period at the poaching price p_{rB}, or buying q_{1B} units in the first period at price p_{1B}, and switching to buy q_{rA} units in the second period at the poaching price p_{rA}. Hence, at an interior solution x_1 must satisfy:

$$v-\frac{1}{\sigma}p_{1A}^{\sigma}-\tau x_1+\delta\left[v-\frac{1}{\sigma}p_{rB}^{\sigma}-\tau(1-x_1)\right]=v-\frac{1}{\sigma}p_{1B}^{\sigma}-\tau\left(1-x_1\right)+\delta\left[v-\frac{1}{\sigma}p_{rA}^{\sigma}-\tau x_1)\right],$$

implying:

$$x_1=\frac{1}{2}+\frac{p_{1B}^{\sigma}-p_{1A}^{\sigma}}{2\sigma\tau\left(1-\delta\right)}+\frac{\delta}{2\sigma\tau\left(1-\delta\right)}\left(p_{rA}^{\sigma}-p_{rB}^{\sigma}\right), \tag{1}$$

in which p_{rA} and p_{rB} are given by the expressions in Proposition 1. Note that when $x_1=\frac{1}{2}$ it follows that $p_{rB}=p_{rA}$ if and only if $p_{1A}=p_{1B}$. First period profits can be written as:

$$\pi_{1A}=p_{1A}D_{1A}=p_{1A}x_1p_{1A}^{\sigma-1}=x_1p_{1A}^{\sigma}, \tag{2}$$

for firm A and:

$$\pi_{1B} = p_{1B} D_{1B} = p_{1B} (1 - x_1) p_{1B}^{\sigma-1} = (1 - x_1) p_{1B}^{\sigma}, \tag{3}$$

for firm B respectively. We are now able to characterize firms' first period problem. Firm A, for example, chooses p_{1A} to maximizes overall profits:

$$\max_{p_{1A}} \pi_A = \pi_{1A} + \delta \pi_{2A},$$

recalling that: $\pi_{2A}(x_1(p_{1A}, p_{1B})) = x_{2A} p_{oA}^{\sigma} + (x_{2B} - x_1) p_{rA}^{\sigma}$. Solving the problem, allows us to state:

Proposition 3 *There is a symmetric Subgame Perfect Nash Equilibrium in which: (i) first-period equilibrium price and quantity purchased are respectively given by*

$$p_1 = \left[\tau\sigma\left(1 + \frac{\delta}{3}\right)\right]^{\frac{1}{\sigma}},$$

$$q_1 = \left[\tau\sigma\left(1 + \frac{\delta}{3}\right)\right]^{\frac{\sigma-1}{\sigma}}.$$

and both firms share equally the market in period 1, thus $x_1(p_A^1, p_B^1) = \frac{1}{2}$*; (ii) second-period equilibrium prices and quantities are:*

$$\begin{aligned} p_o &= \left(\frac{2}{3}\tau\sigma\right)^{\frac{1}{\sigma}}, \quad q_o = \left(\frac{2}{3}\tau\sigma\right)^{\frac{\sigma-1}{\sigma}}, \\ p_r &= \left(\frac{1}{3}\tau\sigma\right)^{\frac{1}{\sigma}}, \quad q_r = \left(\frac{1}{3}\tau\sigma\right)^{\frac{\sigma-1}{\sigma}}. \end{aligned}$$

Proof See Appendix.

5 Competitive effects of price discrimination and demand elasticity

The effects of BBPD *vis à vis* non discriminatory prices can now be evaluated. As previously underlined, inelastic demand is captured in our model as a limiting case when $\sigma = 1$; in that case, our results clearly coincide with the received literature (e.g. Fudenberg and Tirole, 2000). Hence, we shall evaluate the impact of price elasticity, $\sigma \in (0, 1)$, on prices, quantities, switching, profits, consumers' surplus and overall welfare.

We consider prices first. The comparison of the two pricing regimes (no discrimination and BBPD) leads to:

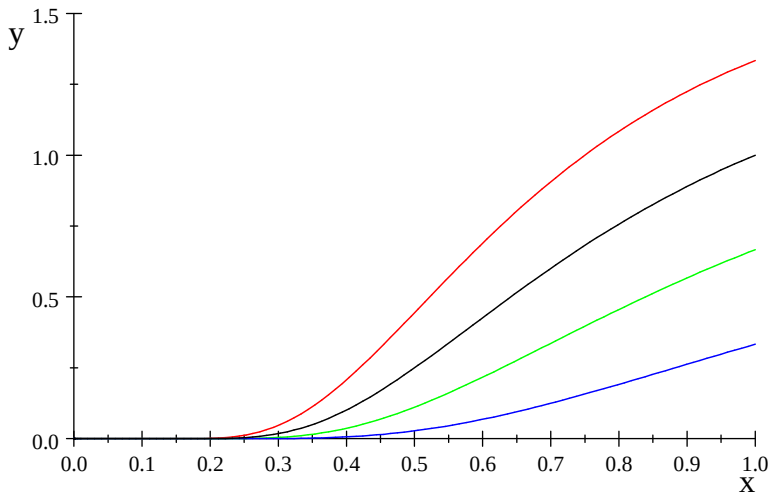
Proposition 4 *(i) The following relationship between first period, second period and non discriminatory prices:*

$$p_r < p_o < p^{nd} \leq p_1$$

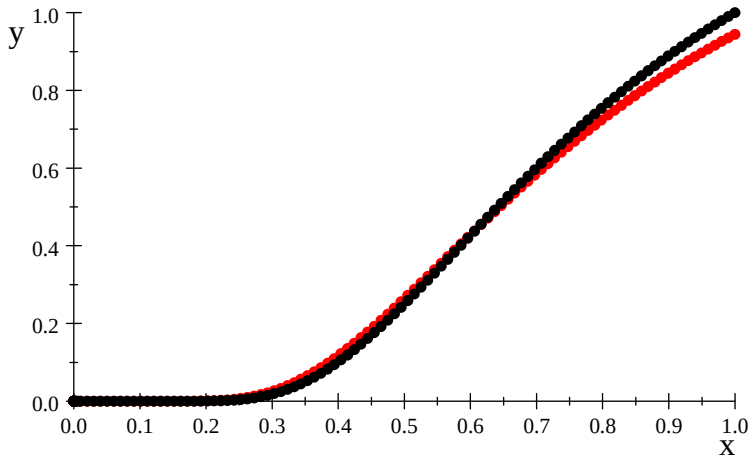
holds no matter the elasticity of demand, $\sigma \in (0, 1]$. *As* $\sigma \to 0$ *(perfectly elastic demand):* $p_r = p_o = p^{nd} = p_1 = 0$. *(ii) Provided that demand is sufficiently elastic and consumers are patient,* $\delta \in (0, 1]$, *the average price paid under BBPD can be higher than the non discriminatory price .*

Proof See Appendix.

The relation between the prices paid by different types of consumers is not affected by elasticity: as in Fudenberg and Tirole (2000), under BBPD consumers are overcharged in the first period but then strong competition leads to reduced prices in the second period. The reduction is more pronounced for switchers that need to be encouraged to buy their less favourite good. Intuitively, the difference between the prices tends to fade out as demand becomes more elastic: in the extreme case of perfect substitutability between the goods, prices tend to the marginal cost, i.e. zero. On average, if demand is inelastic BBPD leads to lower prices; interestingly, however, as the elasticity increases this feature of BBPD may no longer hold: the average price paid under BBPD can exceed the non discriminatory price. The intuition is as follows: as elasticity increases the difference between prices tends to decrease: in that case, the reduction in prices in the second period is not sufficient to compensate for the increase in the first period. The findings are illustrated in Figure 1 and 2 that plot prices as a function of σ.



Prices per type of consumers, $\delta = 1, \tau = 1$. No discrimination (black), first period (red), second period old consumers (green), second period old consumers (blue).



Average prices, $\delta = 1, \tau = 1$. No discrimination (black), BBPD (red).

Prices clearly affect the quantity demanded by each type of consumer and the overall output supplied; this leads to:

Proposition 5 *If demand is elastic, $\sigma \in (0, 1)$: (i) the following relationship between the quantity demanded by each individual consumer in the first*

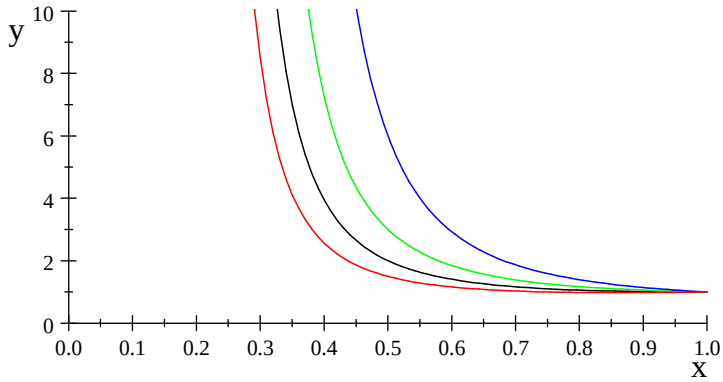
period, second period and under no discrimination (each period) holds:

$$q_1 \leq q^{nd} < q_o < q_r$$

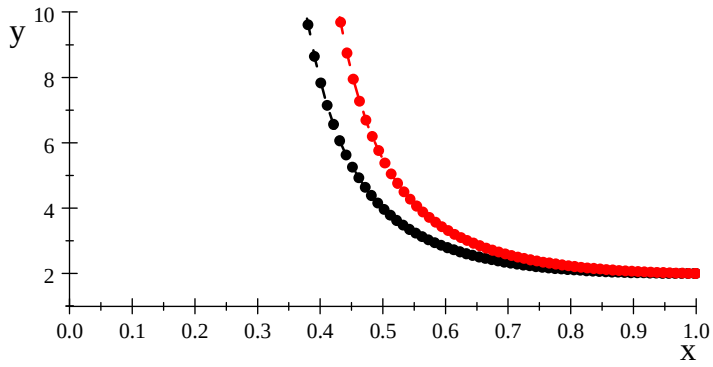
For perfectly inelastic demand, $\sigma = 1$: $q_1 = q^{nd} = q_o = q_r = 1$; (ii) the quantity consumed by any switching consumer over the two periods Q^r exceeds the quantity consumed over the two periods by any loyal consumer Q^o; moreover, Q^o exceeds the quantity consumed over the two periods by any consumer under non discrimination Q^{nd}; (iii) BBPD increases the aggregate quantity consumed in the market.

Proof See Appendix.

In the inelastic benchmark ($\sigma = 1$), any given consumer demands one unit of the good. Elastic demand, instead, implies an inverse relation between price and demand. The consequence is that switching consumers are demanding a higher quantity, both individually and on aggregate through the two periods. Loyal consumers, despite consuming less than switchers, get more of the good than in case discrimination did not take place. This holds both in the second period and over the two periods: under BBPD the effect of a lower price in the second period leads to a demand increase that more than compensates for the higher price and lower consumption in the first period. In aggregate, this implies that BBPD increases overall consumption over the two periods compared with no discrimination. The results are further illustrated in Figure 3 and 4 that plot quantities as a function of σ for $\delta = 1$, $\tau = 1$. $\delta = 1$, $\tau = 1$. In figure 3 no discrimination (black), first period (red), second period old consumers (green), second period new consumers (blue). In figure 4 no discrimination (black), BBPD (red).



Quantity per consumer per period



Overall quantity supplied both periods

We can finally turn to the profit and welfare effects of BBPD.

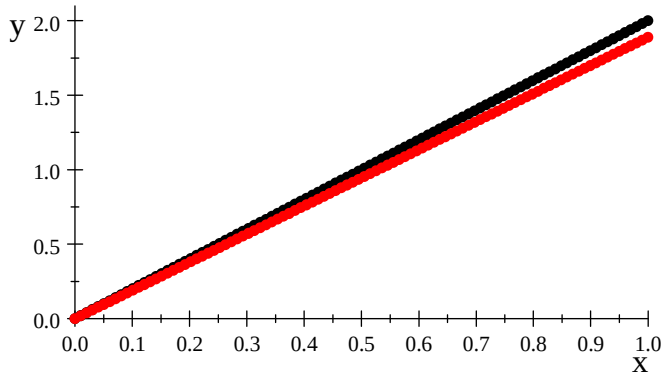
Proposition 6 *(i) Switching and the benefit of consumers under BBPD compared to no discrimination are both unaffected by demand elasticity; (ii) the negative impact of BBPD on profits decreases as demand becomes more elastic; (iii) if demand is sufficiently elastic, BBPD is welfare enhancing.*

Proof See Appendix.

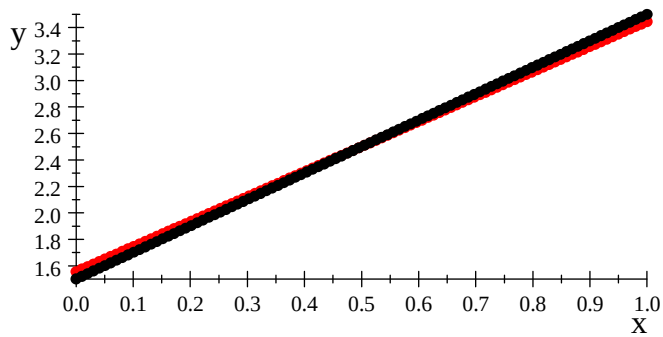
Some of the results of the inelastic benchmark also apply to our framework: the properties of CES preferences are such that both switching and consumers' surplus are not affected by the elasticity of demand. Hence, one third of the total consumers switch to their least favourite firm in the second period; moreover consumers prefer BBPD to no discrimination by. The intuition for the latter result, however, is slightly different. In the inelastic benchmark, under BBPD consumers enjoy lower prices for the same amount of goods consumed as under no discrimination; in our framework, instead, no matter the price effect of BBPD, there is also a demand expansion effect that comes into play. This implies that even if the average price increases under BBPD, the demand expansion effect linked to demand elasticity prevails.

As in the inelastic case, also in our framework BBPD leads to reduced industry profits. The reduction compared to no discrimination, however, becomes lower the more elastic demand is: lower prices imply reduced profit margins under both regimes and, at the extreme of perfectly elastic demand, industry profits tend to zero under both BBPD and no discrimination. This leads to the most interesting result: if the demand is sufficiently elastic, BBPD may actually increase overall welfare. This is in sharp contrast with the inelastic benchmark case. Under inelastic demand, the overall effect of BBPD is just to increase overall transport costs and, hence, reduce the welfare

generated by the market. The result holds if demand is moderately elastic; if elasticity increases enough ($\sigma < 0.5$) the conclusion is instead controverted. The inefficiency created by increased transport costs (i.e. sub-optimal consumption) is more than compensated by the increase in overall consumption induced by the reduced profit margins that firms can charge. The results are illustrated in Figure 5 and 6 that plot overall profits and welfare as a function of σ for $\delta = 1$, $\tau = 1$. No discrimination (black), BBPD (red).



Overall industry profits



Overall welfare for $v = 2$

6 Concluding remarks

This paper constitutes a first assessment of the competitive effects of BBPD when firms face an elastic demand. The results suggest that BBPD can have a positive welfare effect if demand is sufficiently elastic: the increase in transport costs related to switching, that dominates in the standard inelastic

demand framework, is more than compensated by the demand expansion effect implied by the reduced prices and profit margins that firms can charge when demand is elastic. The demand expansion effect, that is obviously overlooked by the standard framework *à la Hotelling*, can play a very relevant role: not only it determines the welfare effects of BBPD just discussed but it also explains why BBPD is beneficial to consumers despite it may lead to a slight increase in the average prices charged over the two periods.

A further contribution of this paper is to provide a closed form solution to a model of competitive BBPD with elastic demand. The assumption of CES preferences is crucial to the goal: it is convenient and elegant but it can also be seen as a limitation of our work. Extending the analysis to different or more general preferences is one of the challenges of future research. Finally, the results were derived in a two period model: a further direction for future research may be to address competitive BBPD with elastic demand in an infinite time horizon.

7 Appendix

Proof of Proposition 2

(i) The relevant demand segments are identified by:

$$x_{2A} = \left(\frac{1}{2} + \frac{p_{rB}^{\sigma} - p_{oA}^{\sigma}}{2\tau\sigma}\right), \qquad (x_1 - x_{2A}) = \left(x_1 - \frac{1}{2} + \frac{p_{oA}^{\sigma} - p_{rB}^{\sigma}}{2\tau\sigma}\right),$$

for firm A and:

$$(1 - x_{2B}) = \left(\frac{1}{2} + \frac{p_{rA}^{\sigma} - p_{oB}^{\sigma}}{2\tau\sigma}\right), \qquad (x_{2B} - x_1) = \left(\frac{1}{2} + \frac{p_{oB}^{\sigma} - p_{rA}^{\sigma}}{2\tau\sigma} - x_1\right)$$

for firm B. At an interior solution it is straightforward to find that the profit segments are:

$$\pi_{oA} = x_{2A} p_{oA}^{\sigma} = \left(\frac{1}{2} + \frac{p_{rB}^{\sigma} - p_{oA}^{\sigma}}{2\tau\sigma}\right)\left(\frac{\tau\sigma\left(2x_1 + 1\right)}{3}\right)^{\sigma}$$

$$\pi_{rA} = \left(x_{2B} - x_1\right) p_{rA}^{\sigma} = \left(\frac{1}{2} + \frac{p_{oB}^{\sigma} - p_{rA}^{\sigma}}{2\tau\sigma} - x_1\right)\left(\frac{\tau\sigma\left(3 - 4x_1\right)}{3}\right)^{\sigma}$$

$$\pi_{oB} = \left(1 - x_{2B}\right) p_{oB}^{\sigma} = \left(\frac{1}{2} + \frac{p_{rA}^{\sigma} - p_{oB}^{\sigma}}{2\tau\sigma}\right)\left(\frac{\tau\sigma\left(3 - 2x_1\right)}{3}\right)^{\sigma}$$

$$\pi_{rB} = \left(x_1 - x_{2A}\right) p_{rB}^{\sigma} = \left(\left(x_1 - \frac{1}{2} + \frac{p_{oA}^{\sigma} - p_{rB}^{\sigma}}{2\tau\sigma}\right)\right)\left(\frac{\tau\sigma\left(4x_1 - 1\right)}{3}\right)^{\sigma}$$

so that the second period profits can be written as:

$$\pi_{2A}\left(x_1\left(p_{1A}, p_{1B}\right)\right) = x_{2A} p_{oA}^{\sigma} + \left(x_{2B} - x_1\right) p_{rA}^{\sigma} \tag{4}$$

$$\pi_{2B}\left(x_1\left(p_{1A}, p_{1B}\right)\right) = \left(1 - x_{2B}\right) p_{oB}^{\sigma} + \left(x_1 - x_{2A}\right) p_{rB}^{\sigma} \tag{5}$$

Standard derivations lead to the prices reported in the proposition.

(ii)

(iii)

Q.E.D.

Proof of Proposition 3

Proof of Proposition 4

(i) Consider first p_1 *and* p^{nd}*. The two prices are identical if and only if* $\delta = 0$*. If* $\delta \in (0, 1]$*, the argument of* p_1 *dominates the one of* p^{nd} *as* $\tau\sigma\left(1 + \delta/3\right) > \tau\sigma$*; applying a monotonically increasing transformation to*

both arguments does not change the relationship so $\forall\sigma \in (0,1]$, $p_1 > p^{nd}$. *The difference between* p^{nd} *and* p^o *is* $\left(1-\left(\frac{2}{3}\right)^{\frac{1}{\sigma}}\right)(\tau\sigma)^{\frac{1}{\sigma}} > 0$, $\forall\sigma \in (0,1]$ *implying* $p^{nd} > p^o$. *A similar argument applies to* p^o *and* p^r, *whose difference is* $\left(\left(\frac{2}{3}\right)^{\frac{1}{\sigma}} - \left(\frac{1}{3}\right)^{\frac{1}{\sigma}}\right)(\tau\sigma)^{\frac{1}{\sigma}} > 0$, $\forall\sigma \in (0,1]$ *implying* $p^o > p^r$. *Finally, it is easy to verify that:*

$$\lim_{\sigma\to 0}\left(\frac{1}{3}\tau\sigma\right)^{\frac{1}{\sigma}} = \lim_{\sigma\to 0}\left(\frac{2}{3}\tau\sigma\right)^{\frac{1}{\sigma}} = \lim_{\sigma\to 0}(\tau\sigma)^{\frac{1}{\sigma}} = \lim_{\sigma\to 0}\left[\tau\sigma\left(1+\frac{\delta}{3}\right)\right]^{\frac{1}{\sigma}} = 0.$$

(ii) As there is no change between the two periods, the average non discriminatory price coincides with p^{nd}. *The average price paid by consumers under BBPD is:*

$$\begin{aligned}\widehat{p}^{bbpd} &= \frac{1}{2}p_1 + \frac{1}{2}\left(\frac{1}{3}p_r + \frac{2}{3}p_o\right)\\ &= \frac{1}{2}\left(\tau\sigma\left(1+\frac{\delta}{3}\right)\right)^{\frac{1}{\sigma}} + \frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{3}\tau\sigma\right)^{\frac{1}{\sigma}} + \frac{2}{3}\left(\frac{2}{3}\tau\sigma\right)^{\frac{1}{\sigma}}\right).\end{aligned}$$

Both $\widehat{p}^{bbpd}$ *and* p^{nd} *are increasing functions of* σ *over the domain. The two prices are clearly identical as* $\sigma \to 0$; *moreover,* $p^{nd} > \widehat{p}^{bbpd}$ *for* $\sigma = 1$. *Provided that* $\delta > 0$:

$$\lim_{\sigma\to 0}\frac{p^{nd}}{\widehat{p}^{bbpd}} = \frac{(\tau\sigma)^{\frac{1}{\sigma}}}{\frac{1}{2}\left(\frac{\tau\sigma(\delta+3)}{3}\right)^{\frac{1}{\sigma}} + \frac{1}{6}\left(\frac{1}{3}\tau\sigma\right)^{\frac{1}{\sigma}} + \frac{1}{3}\left(\frac{2}{3}\tau\sigma\right)^{\frac{1}{\sigma}}} = 0^+,$$

implying that $\widehat{p}^{bbpd} > p^{nd}$ *as* $\sigma \to 0$. *Hence, we can conclude that the two functions intersect for at least one value of* $\sigma \in (0,1)$.

Q.E.D.

Proof of Proposition 5

(i) As demand is inversely related to prices, the results follow from Proposition 4 (i). In particular, as $\sigma \in (0,1)$, *the function* $X^{\frac{\sigma-1}{\sigma}}$ *is decreasing for any value of the argument* X; *hence, for any given value of* σ, *the smaller the argument, the larger* $X^{\frac{\sigma-1}{\sigma}}$. *But then* $\tau\sigma\left(1+\frac{\delta}{3}\right) \geq \tau\sigma > \frac{2}{3}\tau\sigma > \frac{1}{3}\tau\sigma$ *implies* $q_1 \leq q^{nd} < q_o < q_r$. *Finally, it can also be verified that if demand is perfectly inelastic:*

$$\lim_{\sigma\to 1}\left(\frac{1}{3}\tau\sigma\right)^{\frac{\sigma-1}{\sigma}} = \lim_{\sigma\to 1}\left(\frac{2}{3}\tau\sigma\right)^{\frac{\sigma-1}{\sigma}} = \lim_{\sigma\to 1}(\tau\sigma)^{\frac{\sigma-1}{\sigma}} = \lim_{\sigma\to 1}\left[\tau\sigma\left(1+\frac{\delta}{3}\right)\right]^{\frac{\sigma-1}{\sigma}} = 1.$$

(ii) The quantity consumed over two periods by a given switching consumer is:

$$Q^r = \left[\tau\sigma\left(1+\frac{\delta}{3}\right)\right]^{\frac{\sigma-1}{\sigma}} + \left(\frac{1}{3}\tau\sigma\right)^{\frac{\sigma-1}{\sigma}}. \tag{6}$$

The corresponding quantity consumed by a loyal consumer is:

$$Q^o = \left[\tau\sigma\left(1+\frac{\delta}{3}\right)\right]^{\frac{\sigma-1}{\sigma}} + \left(\frac{2}{3}\tau\sigma\right)^{\frac{\sigma-1}{\sigma}}, \tag{7}$$

while any given consumer under no discrimination consumes:

$$Q^{nd} = 2\left(\tau\sigma\right)^{\frac{\sigma-1}{\sigma}}. \tag{8}$$

Focus first on (6) and (7). The first term is identical in both but from point (i) we know that $q_o < q_r$, $\sigma \in (0,1)$. Hence, $Q^r > Q^o$. Turning to (7) and (8), we can write the difference of the two as:

$$\Delta Q = Q^o - Q^{nd} = \left[\underbrace{\left(\tau\sigma\left(1+\frac{\delta}{3}\right)\right)^{\frac{\sigma-1}{\sigma}} - \left(\tau\sigma\right)^{\frac{\sigma-1}{\sigma}}}_{A}\right] + \left[\underbrace{\left(\frac{2}{3}\tau\sigma\right)^{\frac{\sigma-1}{\sigma}} - \left(\tau\sigma\right)^{\frac{\sigma-1}{\sigma}}}_{B}\right],$$

from part (i) we know that $A \leq 0$ while $B > 0$. In case $\delta = 0$ then the result is obvious. If, instead, $\delta \in (0,1]$, we consider once again the function $X^{\frac{\sigma-1}{\sigma}}$; as the function is decreasing in X then $|A| - B = \left(\frac{2}{3}\tau\sigma\right)^{\frac{\sigma-1}{\sigma}} - \left[\tau\sigma\left(1+\frac{\delta}{3}\right)\right]^{\frac{\sigma-1}{\sigma}} < 0$ implying $\Delta Q > 0$.

(iii) The result descends immediately from point (ii). As the market is covered under both no discrimination and BBPD and as both switchers and loyal consumers consume over the two period more than any consumer under no discrimination, then surely BBPD increases the overall quantity consumed, or $Q^{bbpd} = \frac{1}{3}Q^r + \frac{2}{3}Q^o > Q^{nd}$.

Q.E.D.

Proof of Proposition 6

(i) Consumers' switching is independent of the demand elasticity as:

$$S = (x_1^* - x_{2A}^*) + (x_{2B}^* - x_1^*) = \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{2}{3} - \frac{1}{2}\right) = \frac{1}{3}.$$

Turning to consumers' surplus, under no discrimination it is identical in both periods. The overall surplus is then:

$$OCS^{nd} = \left(v - \frac{5}{4}\tau\right)(1+\delta)$$

Overall consumer surplus under BBPD is obtained as:

$$\begin{aligned} OCS^{bbpd} &= 2\left[\int_0^{\frac{1}{2}}\left(v-\left(1+\frac{\delta}{3}\right)\tau-\tau x\right)dx\right] \\ &\quad +2\delta\left[\int_0^{\frac{1}{3}}\left(v-\frac{2}{3}\tau-\tau x\right)dx+\int_{\frac{1}{3}}^{\frac{1}{2}}\left(v-\frac{1}{3}\tau-\tau(1-x)\right)dx\right] \\ &= v\left(1+\delta\right)-\frac{5}{4}\tau-\frac{43}{36}\tau\delta \end{aligned}$$

Both expressions are independent on σ and, clearly, $\forall\delta\in(0,1]:OCS^{nd}<OCS^{bbpd}$.

(iii) The industry profits under no discrimination are:

$$\Pi^{nd}=\left(1+\delta\right)\tau\sigma$$

while under BBPD:

$$\Pi^{bbpd}=\frac{(8\delta+9)}{9}\tau\sigma$$

so the difference is:

$$\Delta\Pi=\Pi^{bbpd}-\Pi^{nd}=-\frac{1}{9}\sigma\tau\delta$$

which is clearly larger (in absolute value), the larger is σ; consequently, the profit differential is decreasing (in absolute value) with the elasticity of demand.

(iii) From the previous points, total welfare under non-discrimination is:

$$W^{nd}=\Pi^{nd}+OCS^{nd}$$

while under BBPD is:

$$W^{bbpd}=\Pi^{bbpd}+OCS^{bbpd}$$

The welfare effect of moving from non-discrimination to BBPD is:

$$\Delta W=W^{bbpd}-W^{nd}=-\frac{1}{18}\tau\delta(2\sigma-1)$$

hence $\Delta W>0$ for $\sigma<\frac{1}{2}$.

Q.E.D.

References

[1] Anderson, S. and A. De Palma (2000), "From local to global competition", European Economic Review, vol.44(3), 423-448.

[2] Armstrong, M. (2006), "Recent Developments in the Economics of Price Discrimination," *in Advances in Economics and Econometrics: Theory and Applications: Ninth World Congress of the Econometric Society,* ed. by R. Blundell, W. Newey, and T. Persson. Cambridge University Press, Cambridge, UK.

[3] Bernhardt, D., Liu, Q. and Serfes, K. (2007), Product Customization, *European Economic Review 51*, 1396-1422.

[4] Caillaud, B. and De Nijs, R. (2011), Strategic Loyalty Reward in Dynamic Price Discrimination, Unpublished manuscript.

[5] Corts, K. (1998), Third-Degree Price Discrimination in Oligopoly: All-Out Competition and Strategic Commitment, *RAND Journal of Economics 29*, 306-323.

[6] Chen, Y. (1997), Paying Customers to Switch, *Journal of Economics and Management Strategy 6*, 877-897.

[7] Chen,Y., Narasimhan, C. and Zhang, Z. (2001), Individual Marketing with Imperfect Targetability, *Marketing Science 20*, 23-41.

[8] Chen, Y. (2005), Oligopoly Price Discrimination by Purchase History, *The Pros and Cons of Price Discrimination*, The Swedish Competition Authority, Stockholm, 101-130.

[9] Chen, Y., and Zhang, Z. (2009), Dynamic Targeted Pricing with Strategic Consumers, *International Journal of Industrial Organization 27*, 43-50.

[10] Chen, Y. and Pearcy, J. (2010), Dynamic Pricing: when to Entice Brand Switching and when to Reward Consumer Loyalty, *RAND Journal of Economics 41*, 674-685.

[11] Esteves, R.B. (2009a), Customer Poaching and Advertising, *Journal of Industrial Economics 57*, 112-146.

[12] Esteves, R.B. (2009b), A Survey on the Economics of Behaviour-Based Price Discrimination, NIPE Working Paper 5/2009.

[13] Esteves, R.B. (2010), Pricing with Customer Recognition, *International Journal of Industrial Organization 28*, 669-681.

[14] Esteves, R.B. and Regiani, C. (2012), Behavior-Based Price Discrimination with Elastic Demand, NIPE Working Paper.

[15] Fudenberg, D. and Tirole, J. (2000), Customer Poaching and Brand Switching, *RAND Journal of Economics 31*, 634-657.

[16] Fudenberg, D. and Villas-Boas, M. (2007), Behavior-Based Price Discrimination and Customer Recognition, In T. Hendershott (ed.), *Handbook on Economics and Information Systems*, North-Holland, Amsterdam.

[17] Gehring, T., and Stenbacka, R. (2004), Differentiation-Induced Switching Costs and Poaching, *Journal of Economics and Management Strategy 13*, 635-655.

[18] Gehring, T., Shy, O and Stenbacka, R. (2011), History-Based Price Discrimination and Entry in Markets with Switching Costs, *European Economic Review 55*, 732-739.

[19] Gehring, T., Shy, O and Stenbacka, R. (2012), A Welfare Evaluation of History-Based Price Discrimination, *Journal of Industry, Competition and Trade 12*, 373-393.

[20] Gu, Y. and T. Wenzel (2009), "A note on the excess entry theorem in spatial models with elastic demand", International Journal of Industrial Organization, vol.27, 567-571.

[21] Gu, Y. and T. Wenzel (2011), "Transparency, price-dependent demand and product variety", Economics Letters, vol.110, 216-219.

[22] Nero, G. (1999), "Full or partial market coverage? A note on spatial competition with elastic demand", Managerial and Decison Economics, vol.20, 107-111.

[23] Rath, K.P. and G. Zhao (2001), "Two stage equilibrium and product choice with elastic demand", International Journal of Industrial Organization, vol.19, 1441-1455.

[24] Stole, L. (2007), Price Discrimination in Competitive Environments, In Armstrong, M. and R. Porter (eds.), *The Handbook of Industrial Organization 3*, Amsterdam: North-Holland.

[25] Taylor, C. (2003), Supplier Surfing: Competition and Consumer Behaviour in Subscription Markets, *RAND Journal of Economics 34*, 223-246.

[26] Thisse, J. and Vives, X. (1988), On the Strategic Choice of Spatial Price Policy, *American Economic Review 78*, 122-137.

[27] Villas-Boas, M. (1999), Dynamic Competition with Customer Recognition, *RAND Journal of Economics 30*, 604-631.

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