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ORBITAL MANOEUVRES WITH SINGLE-INPUT CONTROL

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We study the problem of orbital control under the constraints on the thrust direction for a nonlinear dynamical model. The satellite is equipped with a passive attitude control system providing one-axis stabilization and a propulsion system consisting of one or two thrusters oriented along the stabilized axis. Different applications, such as formation flying and satellite de-orbiting are considered.

INTRODUCTION

Use of small, simple, and inexpensive satellites continuously grows, finding every day more applications for space missions. Therefore studies of simple systems of spacecraft motion control are always of great interest. The capacity to control the orbital motion of a satellite with limited resources can provide new options for large variety of applications.

It is quite usual to assume that the control trust available on board of the satellite can be arbitrary oriented in space. This assumption is valid in case of several thrusters and/or capacity to perform spacecraft reorientation manoeuvres. However, three-axis stabilization requires quite complex ACS. Instead, one can consider the possibility to install only one thruster and to use a (largely available) simple and lightweight attitude control system to stabilize motion of the thrust axis. In this case, one can formulate the problem of orbital control assuming the thrust axis orientation to be known at any moment of time. The important question is then to determine the classes of orbital manoeuvres that can be performed using the above control systems.

Here we consider properties of satellite orbit control systems, assuming that the direction of the thrust axis is given as a function of the satellite orbital position, that is, that the satellite is subject to a single-input control. One can indicate several examples of such systems; here we consider three particular cases. We study a satellite with a passive magnetic attitude control system and one or two thrusters mounted along the axis of the magnet. The other example is a satellite with the thrust axis fixed in the absolute space. Finally, we examine spacecraft equipped with a balloon of variable effective surface area and subject to solar pressure. To study potential use of these control systems we have chosen two promising applications, namely, formation shape maintenance and satellite de-orbiting.

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Applications of the single-input control concept to the problem of formation maintenance have been considered for several missions (see, e.g., Reference $5, 8, 11$). Much research is focused on compensation of the relative drift of satellites caused by the J_2 harmonic of the Earth's gravitational potential. In our previous studies, a possibility to obtain a periodic relative motion of the chief and deputy satellites has been demonstrated for several types of single-input control, including the control oriented along the geomagnetic field and the control along an axis fixed in the absolute space. In each case sufficient controllability conditions have been deduced. In general, these conditions can be formulated as follows: the vector of control direction should have non-zero components both in the orbital plane and along the normal to the orbit. For the control oriented along the geomagnetic field, the existence of a closed trajectory of relative motion with double period has been established for arbitrary initial conditions. We have also proved that the inaccuracy caused by the errors of the linearized dynamical model can be compensated for. Here we consider nonlinear model of system dynamics and show that the relative trajectories can be kept within a vicinity (though not necessarily small) of the reference orbit.

The other problem considered here is spacecraft de-orbiting. The use of thrusters to this end is one of the options, and in this case the resources required for this manoeuvre depend essentially on the available system of motion control. Recently a passive de-orbiting system for high altitude satellite has been proposed.[7] It uses deployable reflective balloon. The de-orbiting occurs thanks to growth of the eccentricity of satellite's orbit.[6] But References 7, 6 deal with equatorial orbits. Here we suggest two active de-orbiting system. One of them uses a thruster oriented along the geo-magnetic field and the other uses a deployable reflective balloon. Both diminish the total energy when this is possible. Such systems are definitely more complex but their capability to operate can be shown for a large set of initial trajectories.

BACKGROUND NOTES

We use the modified equinoctial orbital elements (p, f, g, h, k, L) to describe the position of a satellite. Here p is the semiparameter, L is the true longitude and the other elements are given by

$$\begin{aligned} f &= e\cos(\omega + \Omega), \\ g &= e\sin(\omega + \Omega), \\ h &= \tan\frac{i}{2}\cos\Omega, \\ k &= \tan\frac{i}{2}\sin\Omega, \end{aligned}$$

where e is the orbital eccentricity, i is the orbital inclination, ω is the argument of perigee, and Ω is the right ascension of the ascending node. The differential equations describing the orbital motion are

$$
\begin{aligned}
\dot{p} &= \frac{2p}{w}\sqrt{\frac{p}{\mu}}\Delta_t,\\
\dot{f} &= \sqrt{\frac{p}{\mu}}\left(\Delta_r \sin L + ((w+1)\cos L + f)\frac{\Delta_t}{w} - (h\sin L - k\cos L)\frac{g\Delta_n}{w}\right),\\
\dot{g} &= \sqrt{\frac{p}{\mu}}\left(-\Delta_r \cos L + ((w+1)\sin L + g)\frac{\Delta_t}{w} + (h\sin L - k\cos L)\frac{f\Delta_n}{w}\right),\\
\dot{h} &= \sqrt{\frac{p}{\mu}}\frac{s^2\Delta_n}{2w}\cos L,\\
\dot{k} &= \sqrt{\frac{p}{\mu}}\frac{s^2\Delta_n}{2w}\sin L,\\
\dot{L} &= \sqrt{\mu p}\left(\frac{w}{p}\right)^2 + \frac{1}{w}\sqrt{\frac{p}{\mu}}(h\sin L - k\cos L)\Delta_n,
\end{aligned}
$$

where Δ_r, Δ_t, Δ_n are the accelerations in the radial, tangential and normal directions, respectively.

FORMATION FLYING

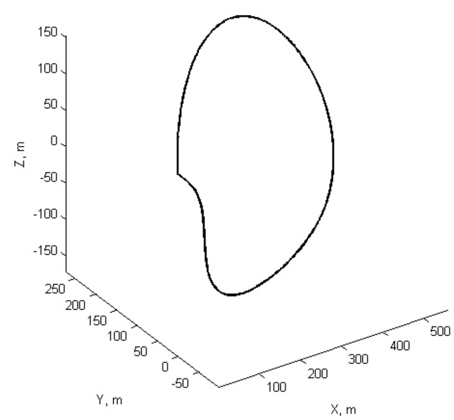

Figure 1. The trajectory of relative motion with bilateral control and magnetic ACS.

The design of formation flying missions is one of the main directions of modern space system development. One of the main problems to be solved in design of a formation flying mission is that of maintenance of the required spatial configuration of satellites. Consider a two-satellite formation the aim of which is to perform measurements or observations, at several points of the orbit. Suppose that the deputy satellite is equipped with a propulsion

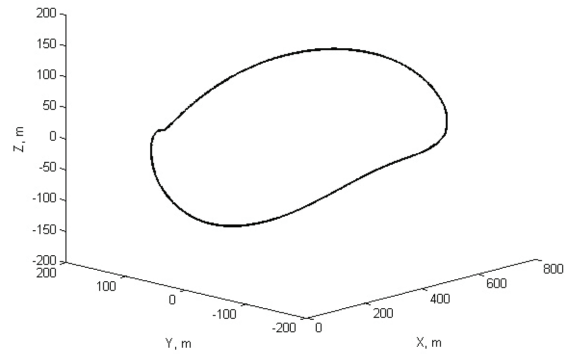

Figure 2. The trajectory of relative motion with bilateral control along an axis fixed in absolute space.

system with its thrust axis fixed in the body of satellite. The thrust can be directed in both ways or in only one, depending on the propulsion system employed.

The Schweighart-Sedwick model

Recall some recent results obtained in Reference 5. The work deals with the general problem of compensation of J_2 perturbations for the deputy satellite in two-satellite formation. The chief satellite is assumed to move passively. The dynamics of the relative motion is governed by the Schweighart-Sedwick linear equations (see Reference 10), a modification of the Hill-Clohessy-Wiltshire equations:

$$\begin{aligned}
&\ddot{x} + 2nc\dot{z} = w(t)e_x(t), \\
&\ddot{y} + q^2 y = 2lq\cos(qt + \phi) + w(t)e_y(t), \\
&\ddot{z} - 2nc\dot{x} - (5c^2 - 2)n^2 z = w(t)e_z(t).
\end{aligned}$$

The linearization is done with respect to the circular reference orbit with the mean motion n. Here x, y, and z are coordinates in the respective orbital reference frame $Oxyz$. The axes are chosen in the following way: Oz indicates the radial direction outwards from the Earth, Ox is directed along the velocity of the point O, and y is normal to the orbital plane. The coefficients c, q, l, and ϕ are properly defined constants. This modification well describes the effect of J_2 perturbations and has been successfully used to study many problems of relative dynamics, such as formation keeping and rendezvous (see, e.g., References 1, 2, 9, 4). Two different types of single-input control have been considered:

1. Bilateral control oriented along a vector fixed in the inertial space (the case of spin stabilization);
2. Bilateral and unilateral control oriented along the vector of local geomagnetic field (the case of passive magnetic stabilization).

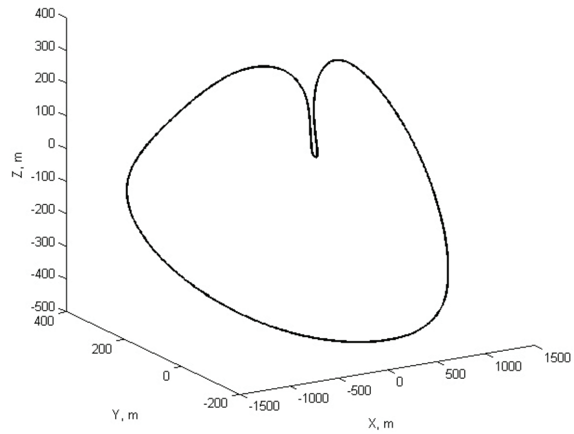

Figure 3. The trajectory of relative motion with unilateral control and magnetic ACS.

It has been shown that for any initial conditions there exists a control that provides a periodic relative motion of chief and deputy satellites with a period T between 1 and 2 orbital periods. This means that the maximum distance between satellites does not become very large. Though the shape of relative trajectory is not controlled, the existence of bounded short-period relative motion suffices to perform the required measurements in many nano- and picosatellite formation missions. Note that in the case of unilateral controls, the situation is more involved.

Nonlinear equations of motion

To treat the nonlinear case we use a Newton-type method recently developed to solve systems of nonlinear equations with geometric constraints (see Reference 3). Let

$$\dot{z} = f(t, z, u), \quad u \in \mathcal{U} \subset R, \quad t \in [0, T], \tag{1}$$

be the system describing the relative motion of two satellites. The initial position is $z(0) = z_0 \in R^6$. Our aim is to find an admissible control $u(\cdot)$ guaranteeing the equality

$$z(T, z_0, u(\cdot)) = z_T \in R^6.$$

The set of controls is $\mathcal{U} = R$ or $\mathcal{U} = R_+ = \{u \mid u \geq 0\}$. To numerically solve the problem consider a discrete-time system

$$z_{k+1} = z_k + \tau f(k\tau, z_k, u_k), \quad u_k \in \mathcal{U}, \quad k = \overline{0, N-1}. \tag{2}$$

Here $\tau = T/N$. Our aim is to find a solution to the system

$$F(Z, U) = 0, \quad U \in \mathcal{U}^N, \tag{3}$$

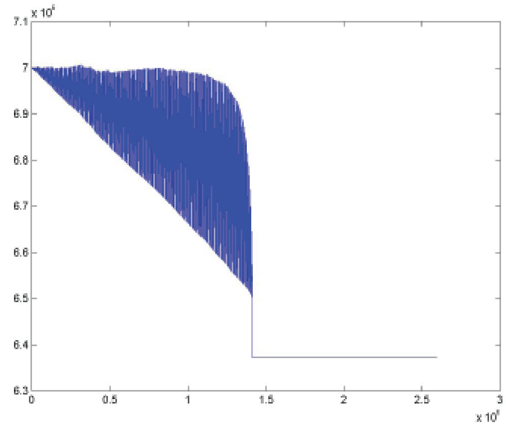

Figure 4. De-orbiting from a near polar orbit with unilateral control and magnetic ACS: the radius of the orbit.

where $F : R^{6(N-1)} \times \mathcal{U}^N \to R^{6N}$ is given by

$$F(Z,U) = \begin{pmatrix} z_1 - z_0 - \tau f(0, z_0, u_0) \\ z_2 - z_1 - \tau f(\tau, z_1, u_1) \\ \cdots \\ z_{k+1} - z_k - \tau f(k\tau, z_k, u_k) \\ \cdots \\ z_T - z_{N-1} - \tau f((N-1)\tau, z_{N-1}, u_{N-1}) \end{pmatrix}.$$

The Newton-type method from Reference 3 takes the following form. Set

$$\Lambda(Z,U)(\bar{Z},\bar{U}) = \nabla_Z F(Z,U)\bar{Z} + \nabla_U F(Z,U)\bar{U},$$

where

$$\nabla_Z F(Z,U)$$
$$= \begin{pmatrix} I_6 & \cdots & 0 \\ -I_6 - \tau \nabla_z f(\tau, z_1, u_1) & \ddots & 0 \\ 0 & \ddots & 0 \\ \cdots & \cdots & \cdots \\ 0 & \cdots & I_6 \\ 0 & \cdots & -I_6 - \tau \nabla_z f((N-1)\tau, z_{N-1}, u_{N-1}) \end{pmatrix},$$

$$\nabla_U F(Z,U)$$
$$= -\tau \begin{pmatrix} \nabla_u f(0, z_0, u_0) & 0 & \cdots & 0 \\ 0 & \nabla_u f(\tau, z_1, u_1) & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & \nabla_u f((N-1)\tau, z_{N-1}, u_{N-1}) \end{pmatrix},$$

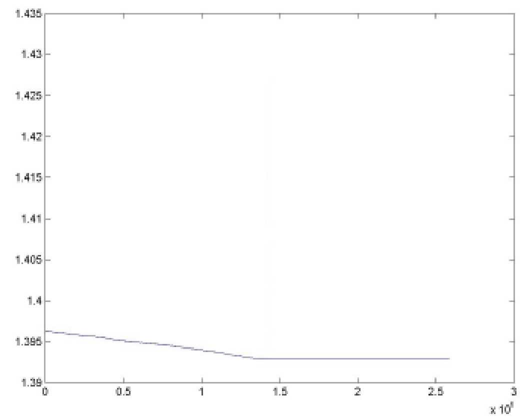

Figure 5. De-orbiting from a near polar orbit with unilateral control and magnetic ACS: the inclination of the orbit.

$\bar{Z} = (\bar{z}_1, \ldots, \bar{z}_{N-1}) \in R^{6(N-1)}$, and $\bar{U} = (\bar{u}_0, \ldots, \bar{u}_{N-1}) \in \mathcal{U}^N$. To find a solution to Problem (3) we apply the following algorithm. Let (Z, U) be given. The next iterate (Z', U') is calculated as follows.

1. Solve the problem

$$\|\bar{Z}\|^2 \to \min, \tag{4}$$
$$-F(Z,U) = \Lambda(Z,U)(\bar{Z},\bar{U}), \tag{5}$$
$$\bar{U} \in \mathcal{U}^N. \tag{6}$$

2. Solve the problem

$$\|F(Z + h\bar{Z}, W)\|^2 \to \min, \tag{7}$$
$$W \in \mathcal{U}^N, \tag{8}$$
$$h \in [0,1], \tag{9}$$

   where $\bar{Z}$ is the solution to the previous problem.

3. Set $Z' = Z + h\bar{Z}$ and $U' = W$, where h and W are solutions to the previous problem.

In Fig. 1 we can see a trajectory of relative motion in the case of bilateral control with magnetic ACS. A trajectory of relative motion in the case of bilateral control along an axis fixed in absolute space is shown in Fig. 2. Finally, in Fig. 3 a trajectory of relative motion in the case of unilateral control with magnetic ACS is shown.

DE-ORBITING

In this section we consider some de-orbiting control algorithms for satellites with unilateral control. We used the atmosphere model from Reference 12.

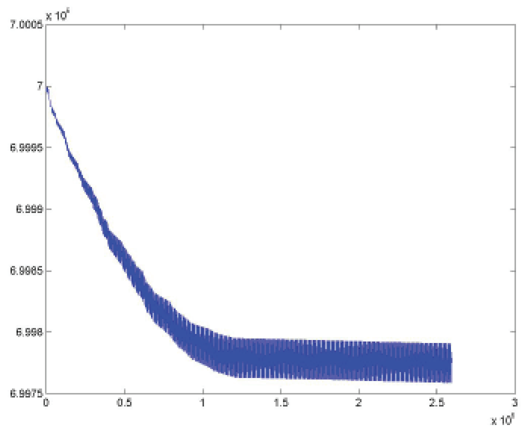

Figure 6. Reaching an equatorial orbit with unilateral control and magnetic ACS: the radius of the orbit.

Thruster oriented along the geomagnetic field

Suppose that the satellite's thruster is oriented along the geomagnetic field. In this case we have

$$\Delta_r = -\frac{2\sin i \sin u}{\sqrt{1+3\sin^2 i \sin^2 u}} F,$$
$$\Delta_t = \frac{\sin i \cos u}{\sqrt{1+3\sin^2 i \sin^2 u}} F,$$
$$\Delta_n = \frac{\cos i}{\sqrt{1+3\sin^2 i \sin^2 u}} F,$$

where $F \geq 0$ is the acceleration and u is the argument of the latitude. The total energy is given by

$$W = -\frac{\mu}{2a},$$

where a is the semimajor axis. Denote by $DW(p, f, g, h, k, L, F)$ the derivative of the total energy along the trajectory of the satellite at the point (p, f, g, h, k, L) with the acceleration F. A natural de-orbiting control algorithm is given by the following rule:

$$F_0 = \begin{cases} F_{\max}, & DW(p, f, g, h, k, L, 1) < 0, \\ 0, & \text{otherwise.} \end{cases}$$

The satellite equipped with such a controller will either reach the Earth's atmosphere and burn up or will move during infinite time in the equatorial plane where the condition $DW(p, f, g, h, k, L, 1) \equiv 0$ is satisfied. Figures 4 - 7 illustrate these situations. The initial orbit of the satellite is a circular one. The radius is 7000 km, and $F_{\max} = 0.001$ m/s^2. If the orbit is rather close to the polar one then we observe a de-orbiting. If the orbit is almost equatorial, the satellite reaches the set where $DW(p, f, g, h, k, L, 1) \equiv 0$.

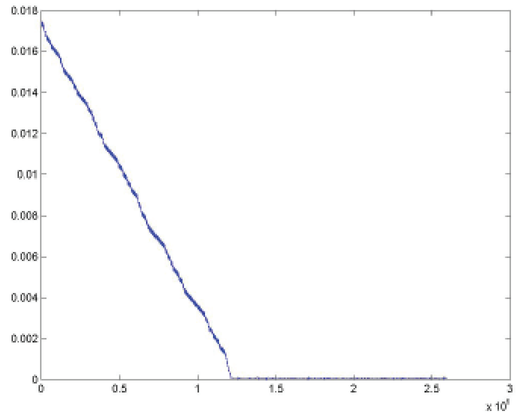

Figure 7. Reaching an equatorial orbit with unilateral control and magnetic ACS: the inclination of the orbit.

The use of solar radiation pressure

Another example of a unilateral control analysed here is the solar radiation pressure. Assume that the satellite is equipped with an inflatable balloon. When the balloon is inflated, the solar pressure achieves its maximal value and if we blow off the balloon the solar pressure is almost zero. This gives us a possibility to control the trajectory of the satellite. Denote by Q the matrix with the columns q_r, q_t, q_n computed from the Earth-centred inertial position r and velocity vectors v according to

$$
\begin{aligned}
q_r &= \frac{r}{\|r\|}, \\
q_n &= \frac{r \times v}{\|r \times v\|}, \\
q_t &= q_n \times q_r.
\end{aligned}
$$

In this case we have

$$
\begin{pmatrix} \Delta_r \\ \Delta_t \\ \Delta_n \end{pmatrix} = -Q^T \begin{pmatrix} \cos\lambda \\ \sin\lambda\cos\epsilon \\ \sin\lambda\sin\epsilon \end{pmatrix} F,
$$

where $\epsilon = 23°27'$ is the angle between the ecliptic plane and the Earth equator, λ is the ecliptic longitude of the Sun, and F is the solar radiation pressure. Again one can use the de-orbiting control algorithm given by

$$
F_0 = \begin{cases} F_{\max}, & DW(p, f, g, h, k, L, 1) < 0, \\ 0, & \text{otherwise.} \end{cases}
$$

An example of de-orbiting is shown in Fig. 8. The initial orbit of the satellite is a circular one with the radius 7000 km. The set where $DW(p, f, g, h, k, L, 1) \equiv 0$ corresponds to a Sun-synchronous orbit with the normal vector pointing towards the Sun.

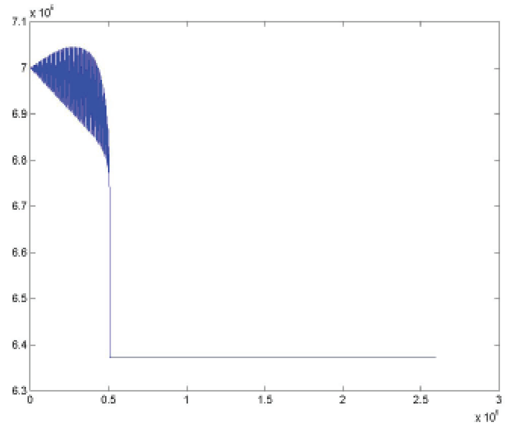

Figure 8. De-orbiting using solar pressure: the radius of the orbit.

CONCLUSIONS

We examine the possibilities for orbit control by means of the trust/propulsion vector when its direction is known as a function of spacecraft location. We consider three types of such single-input control, namely, spacecraft with the thruster oriented along the geomagnetic field, satellite with the thruster axis fixed in the absolute space, and the case of balloon satellite subject to solar pressure. We have shown that single-input control system allow one to perform different and rather complicated orbital manoeuvres, being able, e.g., to keep a satellite in a vicinity of a reference orbit or to de-orbit a satellite in the end of its service.

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