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Abstract. This paper addresses the problem underlying the control and coordination of multiple autonomous airships that must travel maintaining a desired geometric formation and simultaneously avoid collisions with moving or stationary obstacles. The control architecture is based on the attractor dynamics approach to behaviour generation. The airship physical model is presented and the mathematical background for the control architecture is explained. Simulations (with perturbations) with formations of two and three autonomous airships are presented in order to validate the architecture.

1 Introduction

In this paper we address the problem underlying the control and coordination of multiple autonomous airships that must drive maintaining a desired geometric formation and simultaneously avoid collisions with obstacles(e.g. another airship, a building, etc...) see Fig.1. The problem of formation control on land mobile robots has received much attention from researchers working on cooperative robotics (see e.g. [1], [2], [3], [13], [5], [6] and [7] for some interesting works). Research on UAV formation control as also been a subject of growing research(e.g. [8], [9], [10]). In respect to airships, some work is being done on loose formations of stratospheric airships that will serve as telecommunications relays and airborne radar stations(e.g. Lockheed Martin High Altitude Airship).

This project is the next step after the group work on semi-autonomous airships control [11] [12], where we presented a control architecture based on the *dynamical systems approach to behavior generation*(see c.f [18][20][19]). In [4], [13] and [14] the control of formations of land mobile robots was adressed and studied.

Here we show how a set of decentralized and distributed basic control architectures for line, column and "oblique" formations can be used for teams of two airships. These dynamic control architectures can then be easily combined to generate more complex geometric formations for larger teams of airships. As examples we show teams of 3 airships flying in line, column and V formation. We demonstrate the flexibility of our dynamic control architectures, presenting the ability to avoid sensed obstacles integrated with movement in formation. Although we present examples for formations for teams of 3 airships, more complex general configurations (larger number of airships) can be solved by our approach.

We assume that the airships have no prior knowledge of the environment and we follow a master-referenced strategy for each airship in the team. The control architecture of each airship is structured in terms of elementary behaviors (i.e. obstacle avoidance and “keep formation” behavior). The individual behaviors and their integration are modelled by non-linear dynamical systems and bifurcations are used to make design decisions around points at which a system must switch from one type of solution to another. The advantage is that the mathematical properties associated with the concepts (c.f. section 3) enable system integration including stability of the overall behavior of the autonomous systems. The dynamical systems that govern the behavior of each airship are tuned so that the movement of each airship in time is generated as a time series of attractor (i.e. asymptotically stable) states. The benefit is that asymptotical stability can be maintained and thus the systems are robust against environmental perturbations.

The rest of the paper is structured as follows; section 2 describes the background, airship model and the system disturbances. In section 3, we explain how the formations are achieved and maintained during flight. the basic configurations in line, column and oblique are explained and how they can be combined in larger and more complex formations. Section 4 reports on the simulation results on various scenarios with the airship formations avoiding obstacles and changing formations. The last section reports on the conclusions and some facts pertinent to the final results.

2 Airship model and perturbations

The goal is to enable a team of lighter-than-air vehicles to autonomously navigate in formation toward a target destination, avoiding obstacles and coping with environmental perturbations. We briefly discuss the organization of the team of mobile airships and we outline the basic assumptions behind this work.

A team of N airships has one designated *Lead airship* labelled A_1 (the notion of a *Lead airship* is in analogy with the work of Desai, Ostrowsky and Kumar[2]). This airship navigates from an initial position to a final goal destination. Within the formation, each airship (except the *Lead airship*) depends on one of the others. Thus there are many *leaders* and many *followers* but a unique *Lead airship*. We decompose the team of N airships into $N - 1$ sub-teams of 2-airships each (Fig. 1). The control of each sub-team follows a *leader-follower* decentralized motion control strategy (c.f. example in Fig. 5).

Each *follower* airship takes its *leader* as a reference point and its motion must be controlled in order to fulfill the following task requirements (see Fig. 2): *i*) To maintain a desired relative angle between the *leader* and the *follower*, $\Delta\gamma_{i,d}$; *ii*) maintain a desired distance to the *leader*, $l_{i,d}$; *iii*) maintain a desired altitude to the leader, $\Delta h_{i,d}$; and *iv*) simultaneously avoid collisions with obstacles that may appear.

2.1 Airship Kinematics

Each airship is a balloon in which the lift is independent of flight speed, what is called aerostatic lift. Its kinematics description is based on the reference frames presented in figure 3.

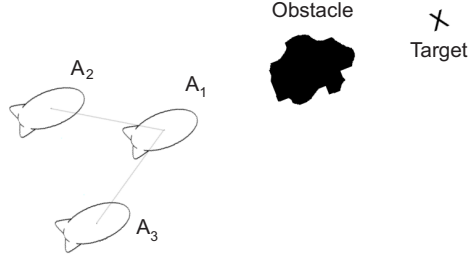


Fig. 1. Example of a team with $N(=3)$ airships in formation. A_1 is the Lead Airship, A_2 and A_3 follows A_1 in an oblique formation.

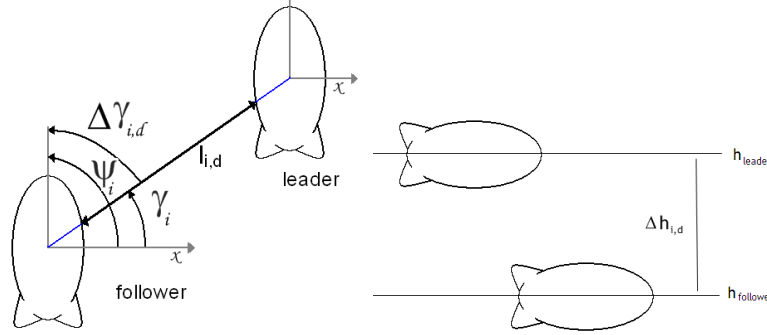


Fig. 2. a) $\Delta\gamma_{i,d}$ is the desired relative angle between the follower airship and its leader. $l_{i,d}$ is the desired distance of the follower to the leader. γ_i is the angle at which the follower sees the leader. b) $\Delta h_{i,d}$ is the desired altitude relative to the leader airship.

We use the following notation: The generalized coordinates for the airship are

$$\eta = (x, y, z, \phi, \theta, \psi)^T \in \mathfrak{R}^6 \quad (1)$$

where $(x, y$ and $z)$ denote the position of the centre of mass, relative to the earth-fixed reference frame, and (ϕ, θ, ψ) are the three Euler angles (i.e. roll, pitch and yaw angle) and represent the orientation of the airship (see [15], [16] or [12]). Therefore, the model partitions naturally into translational and rotational coordinates

$$\eta_1^T = (x, y, z)^T \in \mathfrak{R}^3 \quad \eta_2^T = (\phi, \theta, \psi)^T \in \mathfrak{R}^3 \quad (2)$$

The linear and angular velocity vector with coordinates in body-fixed reference frame- $\{w\}$ (see Fig.3) is

$$v = (v_x, v_y, v_z, \omega_x, \omega_y, \omega_z)^T \in \mathfrak{R}^6 \quad (3)$$

which can be decomposed into:

$$v_1^T = (v_x, v_y, v_z)^T \in \mathfrak{R}^3 \quad v_2^T = (\omega_x, \omega_y, \omega_z)^T \in \mathfrak{R}^3 \quad (4)$$

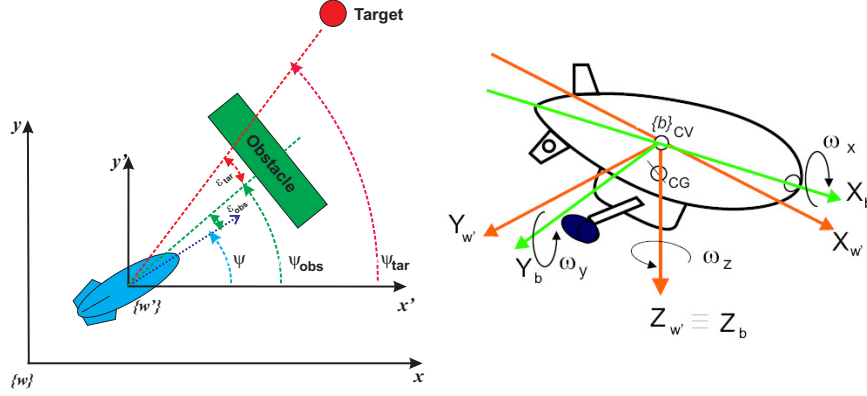


Fig. 3. a) Desired task for the lead airship is movement toward the target location while avoiding obstacles. Constraints for the yaw control (i.e. ψ) are the directions at which obstacles and target lie as seen from the current position of the airship, i.e. ψ_{obs} and ψ_{tar} . Obstacle challenges the movement toward the target location. ϵ_{obs} and ϵ_{tar} are given by the vision system. b) We define three coordinate frames: i) Earth-fixed reference frame $\{w\}$ (X_w, Y_w, Z_w); ii) moving coordinate frame b (X_b, Y_b, Z_b) fixed to the airship and origin coincident with the centre of gravity (CG) (i.e. body-fixed) and iii) w (X_w, Y_w, Z_w) is simply a translation of the earth-fixed reference frame w to the airship's centre of gravity.

The airship's flight path relative to the earth-fixed coordinate system is given by a velocity matrix transformation:

$$\eta_1 = J_1(\eta_2)v_1 \quad (5)$$

The body-fixed angular velocity vector v_2 and the Euler rate vector $\eta_2 = (\dot{\phi}, \dot{\theta}, \dot{\psi})$ are related through a transformation matrix according to:

$$\eta_2 = J_2(\eta_2)v_2 \quad (6)$$

For further details on the Jacobians $J_1(\eta_2)$ and $J_2(\eta_2)$ please refer to [16].

2.2 Airship Dynamics

The airship dynamics can be expressed by the following nonlinear dynamic equation of motion:

$$M\dot{v} + C(v)v + D(v)v + g(\eta) = \tau \quad (7)$$

where variables are described in the reference frame of airship $\{w'\}$ and τ is the vector with the control inputs, i.e. forces (F_x, F_y, F_z) and torques (N_x, N_y, N_z):

$$\tau = (\tau_1^T, \tau_2^T) \quad (8)$$

$$\tau_1^T = [F_x, F_y, F_z]^T \quad (9)$$

$$\tau_2^T = [N_x, N_y, N_z]^T \quad (10)$$

The matrix M is the mass matrix (including added mass terms), C is the Coriolis and centrifugal forces matrix, D is the aerodynamic damping matrix, g is the restoring force (gravity and buoyancy)

In order to get a more realistic behaviour we use the non-linearized model of the airship making the airship model time dependent. This means that C is:

$$C = \begin{bmatrix} 0 & 0 & 0 & mz_G\omega_z & -m(x_G\omega_y - v_z) & m(x_G\omega_z + v_y) \\ 0 & 0 & 0 & -mv_z & m(z_G\omega_z + x_G\omega_x) & mv_x \\ 0 & 0 & 0 & -m(z_G\omega_x - v_y) & -m(z_G\omega_y + v_x) & mx_G\omega_x \\ -mz_G\omega_z & mv_z & m(z_G\omega_x - v_y) & 0 & I_{zz}\omega_z & -I_{yy}\omega_y \\ m(x_G\omega_y - v_z) & -m(z_G\omega_z + x_G\omega_x) & m(z_G\omega_y + v_x) & -I_{zz}\omega_z & 0 & I_{xx}\omega_x \\ m(x_G\omega_z + v_y) & -mv_x & -mx_G\omega_x & I_{yy}\omega_y & -I_{xx}\omega_x & 0 \end{bmatrix} \quad (11)$$

It is not in the scope of this paper to go into further details on this (and also because space is limited), but if you wish to further explore these items please check [15][16][17]. The comprehensive non-linearized airship model is:

$$\dot{x}_{xy} = \begin{bmatrix} -M_{xy}^{-1}(D_{xy} + C_{xy}) & -M_{xy}^{-1}g_{xy} \\ J_{xy} & [0]_{3 \times 3} \end{bmatrix} x_{xy} + \begin{bmatrix} M_{xy}^{-1}B_{xy} \\ [0]_{3 \times 2} \end{bmatrix} u_{xy} \quad (12)$$

$$\dot{x}_{xz} = \begin{bmatrix} -M_{xz}^{-1}(D_{xz} + C_{xz}) & -M_{xz}^{-1}g_{xz} \\ J_{xz} & [0]_{3 \times 3} \end{bmatrix} x_{xz} + \begin{bmatrix} M_{xz}^{-1}B_{xz} \\ [0]_{3 \times 2} \end{bmatrix} u_{xz} \quad (13)$$

The general mass matrix is simplified for the xy and xz system, the same is true for the entire matrix presented in the above systems.

The perturbed state variables for the heading direction (xy) matrix are $x_{xy} = (v_y(t), \omega_x(t), \omega_z(t), y(t), \psi(t), \phi(t))^T$ and the system input is $u_{xy} = Fy$; while for the xz matrix they are $x_{xz} = (v_x(t), v_z(t), \omega_x(t), x(t), z(t), \theta(t))^T$ and $u_{xz} = (Fx, Fz)^T$.

2.3 Environmental Disturbances

In order to test the robustness of the controller design we simulate the behaviour of the airship with perturbations acting along the flight path, the perturbations are added to the corresponding simulink models taking into account the Jacobean matrixes used (see [12] for further details). Two types of environmental perturbations are simultaneously considered and simulated: turbulence and wind. Both contributions were limited to a maximum of one quarter of the airship thrusters maximum torque.

Turbulence By definition turbulence is a state of fluid flow in which the instantaneous velocities exhibit irregular and apparently random fluctuations so that in practice only statistical properties can be recognized and subjected to analysis. With this in mind a perturbation model with stochastic white noise properties was used. This perturbation affects the airship along the longitudinal and translational axis (see Fig. 4, panel A).

Wind The second perturbation used is a gust wind. The wind model that we used can be found in the Matlab Simulink aerospace block set. The mathematical form is as follows:

$$u_w = W_6 \frac{\ln \frac{z}{z_0}}{\ln \frac{20}{z_0}}, 1 \leq z \leq 300m \quad (14)$$

where u_w is the mean wind speed, W_6 is the measured wind speed at an altitude of 6m, z is the airship altitude, and z_0 is a constant equal to 0.0045 for Category C flight phases and 0.18m for all other flight phases. We considered phase C (terminal flight) due to the fact that this is the one that we feel is more adequate to a slow moving airship (see Fig. 4, panel B). The effect of the wind on the airship frame is obtained through the use of the corresponding Jacobean matrixes: J_1 for the heading system and J_2 for the translation control system (see [11]).

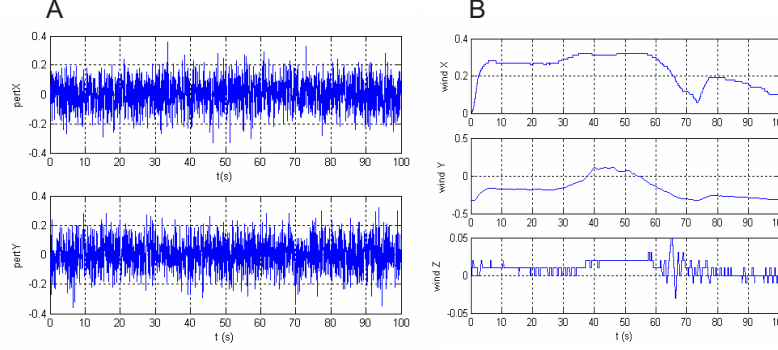


Fig. 4. In panel A we can see a stochastic perturbation acting. In panel B we can see the wind. These are the plots of the perturbations that acted on the airships during the simulation in Fig. 8

3 building airship formations

Our approach is based on the so called Dynamic Approach to Behaviour Generation ([18][19][20]). To model the airships flight behaviour we use its heading direction ψ (i.e. yaw), forward velocity, $v_{x,b}$, and altitude, z . Behaviour is generated by continuously providing values to these variables, which control then the airships motors. The time course of each of these variables is obtained from (fixed point) solutions of dynamical systems. The attractor solutions (asymptotically stable states) dominate these solutions by design. In the present design the controller that governs the behavioural dynamics of $\psi(t)$, $v_{x,b}(t)$ and altitude $z(t)$ is defined as a set of differential equations:

$$\begin{aligned} \dot{\psi}_i &= f(\psi_i, parameters) \\ \dot{v}_{x,b,i} &= g(v_{x,b,i}, parameters) \quad i = follower1, follower2, \dots \\ \dot{z}_i &= h(z_i, parameters) \end{aligned} \quad (15)$$

Task constraints define contributions to the vectors fields f , g and h . The complete control architecture for the trajectory generation of the lead airship is described in [12]

in detail and we will not enter into any particular details in this paper. Next we build the vector fields which erects an attractor at $z_{leader} + \Delta h_{i,d}$ with relaxation

$$\dot{z}_i = -\lambda_z(z_i - z_{leader} + \Delta h_{i,d}) \quad \text{with} \quad \lambda_z > 0 \quad (16)$$

where z_{leader} is the targets altitude and defines the desired value for the airships altitude and λ_z is the relaxation rate.

Now, consider two airships that navigate in a world, keeping the distance between them constant. Then, we state that they are either in a *column* formation, if one is exactly behind the other (see figure 5.a)), or in a *line* formation, if they navigate side-by-side (see figure 5.c)), or in an *oblique* formation, otherwise (see figure 5.b)).

From this set of basic two airship formations, more complex ones can be derived, as we will see later in section 3.4. Next, in sections 3.1 to 3.3 we present the control architecture for each of these two airship formations.

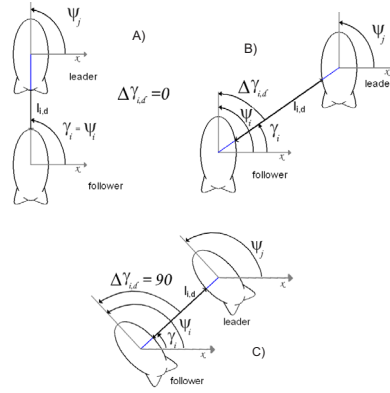


Fig. 5. Possible formation for teams with only two airships. Note that i will refer to the leader and j to the follower(s). The airships can either be in a) column formation; b) oblique formation; c) line formation. The heading direction of the leader and the follower are, respectively, ψ_i and ψ_j . γ_i is the direction at which the follower sees the leader. $l_{i,d}$ is the desired distance between both airships. $\Delta\gamma_{i,d}$ is the desired difference between the followers heading and the direction at which sees the leader.

3.1 Two airships in column

A dynamical system that causes a follower airship to navigate in column formation, maintaining a constant distance, with its leader is:

$$\dot{\psi}_i = f_{col,i} = -\lambda_{col} \sin(\psi_i - \gamma_i) \quad (17)$$

This dynamical system ensures that the airship steers to the desired heading direction, ψ_i (the direction at which the follower sees its leader), by making it an asymptot-

ically stable state of the system. Parameter $\lambda_{col} (> 0)$ is the strength of attraction to the attractor and corresponds to the relaxation rate.

Path velocity is controlled to ensure that the follower adequates its velocity to the leader's one, while trying to maintain the desired distance to it. This is accomplished by making the value of the desired velocity equal to

$$v_{i,d} = \begin{cases} v_j - (l_{i,d} - l_i)/T_{2c} & \text{if } l_i \geq l_{i,d} \\ -v_j - (l_{i,d} - l_i)/T_{2c} & \text{else} \end{cases} \quad (18)$$

T_2C is a parameter that smoothes the airship movement, by controlling its accelerations and decelerations.

3.2 Two airships in oblique

A dynamical system that causes a follower airship to navigate in an oblique formation, maintaining a constant distance and relative orientation, with its leader is:

$$\begin{aligned} \dot{\psi}_i &= f_{oblique}(\psi_i) \\ &= f_{attract}(\psi_i) + f_{repel}(\psi_i) \end{aligned} \quad (19)$$

where each term defines an attractive force ($k = \text{attract, repel}$)

$$f_k(\psi_i) = -\lambda_{oblique}\lambda_k(l_i)\sin(\psi_i - \gamma_k) \quad (20)$$

where the first contribution, $f_{attract}$, erects an attractor at a direction

$$\gamma_{attract} = \gamma_i + \Delta\gamma_{i,d} - \pi/4 \quad (21)$$

The strength of this attractor ($\lambda_{oblique}\lambda_{attract}(l_i)$ with $\lambda_{oblique}$ fixed), increases with distance, l_i , between the two airships:

$$\lambda_{attract}(l_i) = 1/(1 + \exp(-(l_i - l_{i,d})/\mu)). \quad (22)$$

The second contribution, f_{repel} , sets an attractor at a direction pointing away from the leader,

$$\gamma_{repel} = \gamma_i + \Delta\gamma_{i,d} + \pi/4 \quad (23)$$

with a strength ($\lambda_{oblique}\lambda_{repel}(l_i)$) that decreases with distance, l_i , between the airships,

$$\lambda_{repel}(l_i) = 1 - \lambda_{attract}(l_i). \quad (24)$$

The attractor location of the resultant vector field, is thus dependent on the distance between the two airships. When the distance between the two airships is larger than the desired distance the attractive force erected at direction $\gamma_{attract}$ is stronger than the attractive set at direction γ_{repel} . Their superposition leads to an attractor at a direction still pointing towards the movement direction of the leader airship. Conversely, when the distance between the two airships is smaller than the desired distance, the reverse holds, i.e. the attractive force set at direction $\gamma_{attract}$ is now stronger than the attractive force at direction γ_{repel} . The resulting oblique formation dynamics exhibits an attractor

at a direction pointing away from the leader's direction of movement. When the airships are at the desired distance the two attractive forces have the same strength which leads to a resultant attractor at the direction $\gamma_{i,d} = \gamma_i + \Delta\gamma_{i,d}$. Path velocity is controlled exactly in the same way as for column formation.

3.3 Two airships in Line

A dynamical system that causes a follower airship to navigate in a line formation, maintaining a constant distance, with its leader is similar to the one of oblique formation. The only difference lies in $\Delta\gamma_{i,d}$, which is fixed and equal $\pm\pi/2$ depending on the follower driving on the right or left of the leader.

In line formation, the path velocity does not depend only on the distance and velocity of the leader, but we also have to take into account the heading direction of the leader and the direction at which it is seen by the follower. A set of heuristic rules have been written that make the follower accelerate or decelerate depending on the leader's pose relative to the follower:

$$\begin{aligned} v_{i,d,line} = & DE_1 \cdot v_j(1 - |\sin(\gamma_i)|) + \\ & + DE_2 \cdot v_j(1 - |\cos(\gamma_i)|) + \\ & + AC_1 \cdot v_j(1 + K_v |\sin(\gamma_i)|) + \\ & + AC_2 \cdot v_j(1 + K_v |\cos(\gamma_i)|) \end{aligned} \quad (25)$$

where DE_1 , DE_2 , AC_1 and AC_2 are mutually exclusive activation variables that embed the relative attitude of the follower airship regarding the leader. They are set and reset by testing the direction at which the leader is seen by the follower and the heading direction of the leader (see [13] for details).

3.4 N-airship formations

Teams of airships with more than two airships are built by specifying pairs of leader-follower teams and stating the particular configuration to achieve. A complete team specification is accomplished by means of a *formation matrix*:

$$\mathbf{S} = \begin{pmatrix} L_1 & \Delta\gamma_{1,d} & l_{1,d} & \Delta h_{1,d} \\ L_2 & \Delta\gamma_{2,d} & l_{2,d} & \Delta h_{2,d} \\ \dots & \dots & \dots & \dots \\ L_N & \Delta\gamma_{N,d} & l_{N,d} & \Delta h_{n,d} \end{pmatrix} \quad (26)$$

For a team of N airships, where each airship is identified by a specific identification number, the formation matrix has N rows and four columns. Row i relates to the airship with identification number i . The contents of the columns specify the values that characterize a formation, $l_{i,d}$ (cm) and $\Delta\gamma_{i,d}$ (rad) in columns two and three, respectively, the identification number of this airships leader and the fourth column, $\Delta h_{i,d}$ (cm), is the altitude difference to the leader airship. The lead airship is identified by having its row with $l_{i,d} = 0$ and $\Delta\gamma_{i,d} = 0$, while the third column is the distance it should stop from the target and the fourth column is the desired altitude for the formation(see Fig. 6 for an example).

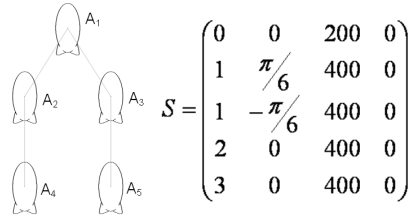


Fig. 6. Example of a formation. airship A_1 is the *Lead airship*, airship A_2 follows A_1 on the left side and maintaining an oblique formation, airship A_3 follows A_1 on the right side and maintaining an oblique formation. airship A_4 follow airship A_2 in a column formation. airships A_5 follow airship A_3 maintaining a column formation.

4 results

In figure 7 we can see a simulation run where the possible configurations are depicted. The airships change from line formation to column formation after reaching a waypoint. At the next waypoint the formation changes to a V formation and at the final waypoint they change to an oblique formation where the airships are at different altitudes

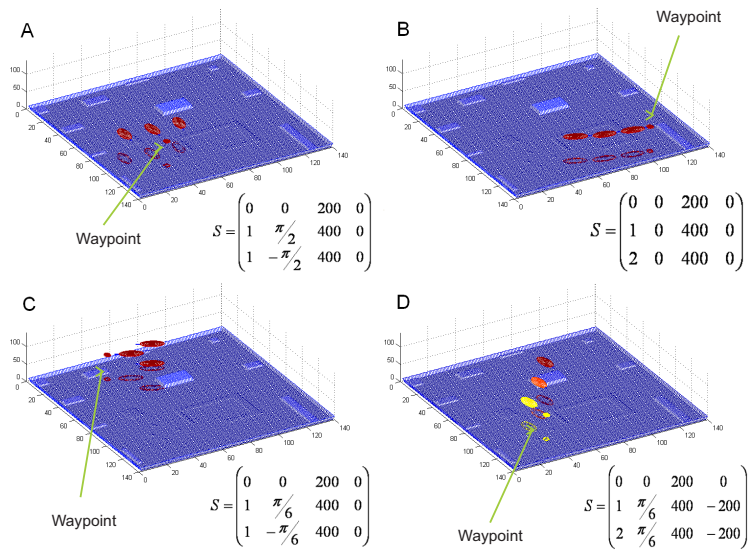


Fig. 7. Simulations of the possible configuration with three airships. In panel A we can see the airships in a line formation, in B the airships are in a column formation, in C in V formation and in D in oblique formation with the airships at different altitudes.

In figure 8 we can observe the airships moving in a cluttered environment maintaining formation (as close as possible) and avoiding obstacles. Depicted here is a case

similar to the one that is depicted in figure 1. The target is placed behind an obstacle and the *Leader* airship travels to that position. The *Followers* airships follow the master in a oblique formation avoiding obstacles and keeping formation.

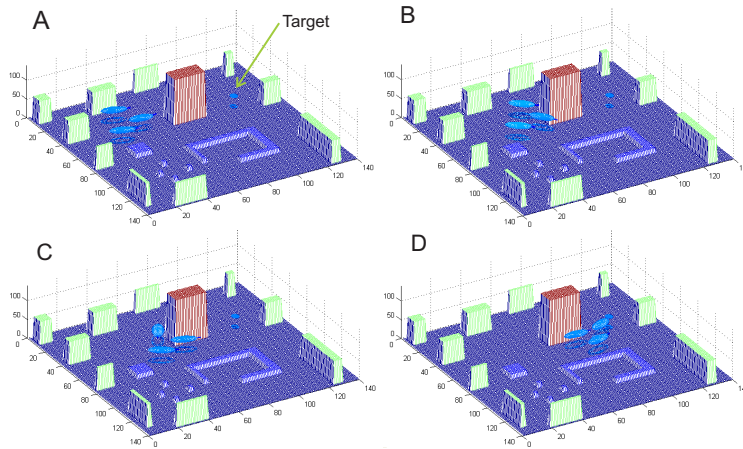


Fig. 8. In panels A through D we can see the airship formation traveling to a target position while avoiding an obstacle in its flight path.

5 Conclusions

We demonstrated that the formation control architecture is capable of controlling n-airship formations using simple behaviours. These simple behaviours are the column, line and oblique formations of two airships. All N-airship formations can be "built" from these simple behaviours. These behaviours enable the formation to avoid stationary obstacles.

The simulation efforts were conducted in the presence of perturbations on the main axis of motion. Although the model of the airship is inertial, we must note that the formation control is done at a kinematic level, where the *behaviour based approach to non-linear dynamical systems* generates the next reference point for the airship control variables. It is expected that the airship actuators are capable of "following the orders" given by the dynamical system. The actuator model is also taken into account during the simulation, nevertheless, the dynamical system does not generate forces or torques to be applied directly, it generates reference values to be followed by the robot. As with everything this has advantages and disadvantages. The main advantage is the fact that, with some parameters adjustments, the control architecture can be used to control any kind of airship - the control architecture is not model dependent. On the other hand, it is expected that the physical platform be able to perform adequately to the references provided.

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