Abstract - This paper deals with problems related with harmonics in power system networks. Several international standards issued to control power quality problems are briefly described and some important methods to analyse electrical circuits with non-sinusoidal waveforms are introduced and evaluated. One of these methods - the p-q theory - was used to implement the control algorithm of a shunt active filter, which is also described in this paper as an application example. The filter can compensate for harmonic currents, power factor and load unbalance. Both simulation and experimental results are presented, showing that good dynamic and steady-state response can be achieved with this approach.

Index Terms -- Active Filters, Harmonics Compensation, Power Factor Correction, Power Quality.

I. INTRODUCTION

Due to the intensive use of power converters and other non-linear loads in industry and by consumers in general, it can be observed an increasing deterioration of the power systems voltage and current waveforms. Figure 1 presents a power system with sinusoidal source voltage ($v_s$) operating with a linear and a non-linear load. The current of the non-linear load ($i_{L1}$) contains harmonics. The harmonics in the line-current ($i_s$) produce a non-linear voltage drop ($\Delta v$) in the line impedance, which distorts the load voltage ($v_L$). Since load voltage is distorted, even the current at the linear load ($i_{L1}$) becomes non-sinusoidal.

The presence of harmonics in power lines results in greater power losses in the distribution system, interference problems in communication systems and, sometimes, in operation failures of electronic equipments, which are more and more sensitive since they include microelectronic control systems, which work with very low energy levels.

Because of these problems, the issue of the power quality delivered to the end consumers is, more than ever, an object of great concern.

International standards concerning electrical power quality (IEEE-519, IEC 61000, EN 50160, among others) impose that electrical equipments and facilities should not produce harmonic contents greater than specified values, and also specify distortion limits to the supply voltage. Meanwhile, it is mandatory to solve the harmonic problems caused by those equipments already installed.

These problems can be classified into two kinds: instantaneous effects and long-term effects.

The instantaneous effects problems are associated with interferences, malfunction or performance degradation of equipments and devices.

Long-term effects are of thermal nature and are related, to additional losses and overheating, causing a reduction of the mean lifetime of capacitors, rotating machines and transformers.

Passive filters have been used as a solution to solve harmonic current problems, but they present several disadvantages, namely: they only filter the frequencies they were previously tuned for; their operation cannot be limited to a certain load; resonances can occur because of the interaction between the passive filters and other loads, with unpredictable results. To cope with these disadvantages, recent efforts have been concentrated in the development of active filters (Fig. 2).

II. POWER QUALITY STANDARDS

To assure the harmonization of legislation within the European Community, without which the free interchange of goods and services would be affected, several directives have been released. One of such directives is the Council Directive 85/374, related to the liability for defective products. Its 2nd article defines electricity as product and in this sense it becomes necessary to establish its characteristics.

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J. L. Afonso is with the Department of Industrial Electronics, University of Minho, 4800 058 Guimarães, Portugal (e-mail: jla@dei.uminho.pt).

H. J. R. Silva is with the Department of Electrical Engineering, ISEP - Polytechnic Institute of Porto, 4200 072 Porto, Portugal (e-mail: henrique@dee.isep.ipp.pt).

J. S. Martins is with the Department of Industrial Electronics, University of Minho, 4800 058 Guimarães, Portugal (e-mail: jmartins@dei.uminho.pt).
**A. EN 50160**

“Voltage Characteristics of Electricity Supplied by Public Distribution Systems” – This standard, published by CENELEC (European Committee for Electrotechnical Standardization) defines the main characteristics of the low and medium voltage supplied by public distribution networks at the PCC (point of common coupling) [1].

**TABLE I**

<table>
<thead>
<tr>
<th>Harmonic Voltages at PCC Until Order 25 in Percentage of the Nominal Voltage</th>
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</thead>
<tbody>
<tr>
<td>Non-multiple of 3</td>
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<tr>
<td>-------------------</td>
</tr>
<tr>
<td>Order n</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>7</td>
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<tr>
<td>11</td>
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<td>13</td>
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<td>23</td>
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<tr>
<td>25</td>
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</tbody>
</table>

Note: The values corresponding to harmonics of order greater than 25, in general low and very unpredictable, as a cause of the resonance effects, are not indicated in this table

This standard also establishes that voltage total harmonic distortion (THD), including the first 40 harmonics, must not exceed 8%.

**B. IEC 61000**

This set of IEC (International Electrotechnical Commission) standards [2-4] is concerned with electromagnetic compatibility (EMC) and includes the following parts:

1. General – General considerations, definitions and terminology: 61000-1-x.
2. Environment – Description of the environment, classification of the environment, compatibility levels: 61000-2-x.
3. Limits – Emission limits, immunity limits: 61000-3-x.
4. Testing and Measurement Techniques – Provides techniques and measurement rules in order to assure the compliance with the other parts of the standard: 61000-4-x.
5. Installation and mitigation guidelines – Provides guidelines in the application of equipment such as filters, compensators, surge arresters, etc, in order to solve the problems related with power quality: 61000-5-x.
6. Generic standards – Sets up the required immunity levels for general-purpose equipments or for specific types of equipment: 61000-6-x.
7. Miscellaneous: 61000-9-x.

**TABLE II**

<table>
<thead>
<tr>
<th>Compatibility Levels for Individual Harmonic Voltages in Public Low-Voltage Networks</th>
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</thead>
<tbody>
<tr>
<td>Odd harmonics non-multiple of 3</td>
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<tr>
<td>-------------------</td>
</tr>
<tr>
<td>Order n</td>
</tr>
<tr>
<td>5</td>
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<td>25</td>
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</tbody>
</table>

**TABLE III**

<table>
<thead>
<tr>
<th>Compatibility Levels for Harmonics</th>
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</thead>
<tbody>
<tr>
<td>Class 1</td>
</tr>
<tr>
<td>Total Harmonic Distortion</td>
</tr>
</tbody>
</table>

Class 1 applies to protected networks and it has the lowest lower compatibility levels (lower than that public networks). It concerns the use of devices and equipments very sensitive to electric disturbances, v. g. technological laboratories instrumentation, certain automation and protective equipments, specific computers, etc.

Class 2 applies to PCC and to the internal connecting points in the general industrial environment. It also applies to public networks.

Class 3 is only applicable to internal connection points of the industrial environments. The compatibility level is greater than that of the class 2 for certain disturbances. This class should always be considered whenever one of these conditions is met:

- Most of the loads are fed through converters.
- There are melting machines.
- Large capacity drives are started up very often.
- The loads change rapidly.

**C. ANSI/IEEE 519-1992**

According to this standard, which is presently being updated, the distribution companies are responsible for keeping the quality of voltage in all their systems [5]. This standard sets up the distortion limits for the different voltage levels of the electric networks.

**TABLE IV**

<table>
<thead>
<tr>
<th>Maximum Distortion Levels</th>
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<tbody>
<tr>
<td>Nominal Voltage at PCC $U_a$</td>
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<tr>
<td>---------------------------</td>
</tr>
<tr>
<td>$U_a \leq 69$ kV</td>
</tr>
<tr>
<td>$69$ kV &lt; $U_a \leq 161$ kV</td>
</tr>
<tr>
<td>$U_a &gt; 161$ kV</td>
</tr>
</tbody>
</table>
III. ACTIVE FILTERS

Active filters are special equipments that use power electronic converters to compensate for current and/or voltage harmonics originated by non-linear loads, or to avoid that harmonic voltages might be applied to sensitive loads.

There are basically two types of active filters: the shunt type and the series type. It is possible to have active filters combined with passive filters as well as active filters of both types acting together [6].

Figure 3 presents the electrical scheme of a shunt active filter for a three-phase power system with neutral wire, which, can both compensate for current harmonics and perform power factor correction. Furthermore, it allows load balancing, eliminating the current in the neutral wire. The power stage is, basically, a voltage-source inverter with only a single capacitor in the DC side (the active filter does not require any internal power supply), controlled in a way that it acts like a current-source. From the measured values of phase voltages \( (v_{sa}, v_{sb}, v_{sc}) \) and load currents \( (i_{a}, i_{b}, i_{c}) \), the controller calculates the reference currents \( (i_{sa}^*, i_{sb}^*, i_{sc}^*, i_{in}^*) \) used by the inverter to produce the compensation currents \( (i_{ca}, i_{cb}, i_{cc}, i_{cn}) \). This solution requires 6 current sensors and 4 voltage sensors, and the inverter has 4 legs (8 power semiconductor switches). For balanced loads without 3rd order current harmonics (three-phase motors, three-phase adjustable speed drives, three-phase controlled or non-controlled rectifiers, etc) there is no need to compensate for the current in neutral wire. These allow the use of a simpler inverter (with only three legs) and only 4 current sensors. It also eases the controller calculations.

Figure 4 shows the scheme of a series active filter for a three-phase power system. It is the dual of the shunt active filter, and is able to compensate for distortion in the power line voltages, making the voltages applied to the load sinusoidal (compensating for voltage harmonics). The filter consists of a voltage-source inverter (behaving as a controlled voltage source) and requires 3 single-phase transformers to interface with the power system. The series active filter does not compensate for load current harmonics but it acts as high-impedance to the current harmonics coming from the power source side. Therefore, it guarantees that passive filters eventually placed at the load input will not drain harmonic currents from the rest of the power system.

Another solution to solve the load current harmonics is to use a shunt active filter together with the series active filter (Fig. 5), so that both load voltages and the supplied currents become sinusoidal waveforms.

Shunt active filters are already commercially available, although much research is being done, yet. The series and series-shunt types of active filters are yet at prototype level.

IV. CONTROL METHODS FOR ACTIVE FILTERS

The methods applied to control the active filters are decisive in achieving the goals of compensation, in the determination of the filter power rate, and in their dynamic and steady-state performance. Basically, the different approaches regarding the calculation of the compensation currents and voltages from the measured distorted quantities can be grouped into two classes: frequency-domain and time-domain.

The frequency-domain approach implies the use of the Fourier transform and its analysis, which leads to a huge amount of calculations, making the control method very heavy. In the time-domain approach, the traditional concepts of circuit analysis and algebraic transformations associated with changes of reference frames are used, simplifying the control task.
The three-phase power delivered to a load by the source has the well-known expression:
\[ p_i(t) = v_i(t) \cdot i_i(t) + v_a(t) \cdot i_a(t) + v_c(t) \cdot i_c(t) \] (1)
where \( v_i(t), v_a(t), v_c(t) \) represents the instantaneous load voltages referred to the neutral point, and \( i_i(t), i_a(t), i_c(t) \) are the load instantaneous currents. However, for the given voltages, there is more than one set of currents producing the same instantaneous power. So, what is the optimal set of currents for a given power? One possible answer is the set that minimizes power loss in the line.

On the other hand it is known that for a balanced sinusoidal system, in voltage and current, the instantaneous power is constant and so equal to active power, since this value corresponds to the average value of the instantaneous power. So, the best set of currents can be the one that leads to a constant instantaneous power.

Next, three time-domain approach methods used in the control of shunt active filters are briefly presented and commented.

A. FBD Method
The FBD (Frize-Buchholz-Depenbrock) method, proposed by Depenbrock et al. [7] decomposes the load currents into power components and powerless components. The goal is to compensate all the terms that do not produce currents into power components and powerless components.

\[ \text{power} = \text{powerful terms} + \text{powerless terms} \]

where

\[ P = \text{powerful terms} \]
\[ Q = \text{powerless terms} \]

The FBD (Frize-Buchholz-Depenbrock) method, proposed by Akagi et al. [8] uses the Park transform. The Park transformation of the three-phase voltages and currents in the a-b-c coordinates, obtained from the load phase currents are:
\[ \begin{bmatrix} i_0 \\ i_a \\ i_b \end{bmatrix} = \begin{bmatrix} \frac{2}{\sqrt{3}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\sin \theta & \cos \theta & \cos (\theta + \frac{2\pi}{3}) \\ \sin \theta & \cos \theta & \cos (\theta - \frac{2\pi}{3}) \end{bmatrix} \begin{bmatrix} i_0 \\ i_a \\ i_b \end{bmatrix} \]

With this transformation the current fundamental order part will be found in the DC component of the transformed d-q currents, thus making possible its extraction through the use of low-pass filters, for example.

The instantaneous power is given by the expression:
\[ p(t) = v_a \cdot i_a + v_b \cdot i_b + v_c \cdot i_c \]

In order to minimize line power loss a reactive instantaneous power that must be compensated is defined:
\[ \tilde{q}(t) = \tilde{q}_v \cdot i_v - \tilde{q}_q \cdot i_q \]

The vectorial nature of (7) implies that all the three terms must become zero in order to compensate all reactive instantaneous power:
\[ v_q \cdot i_q = 0 \]
\[ v_d \cdot i_d = 0 \]

C. p-q Theory
This theory, also known as “instantaneous power theory” was proposed in 1983 by Akagi et al. [9, 10] to control active filters. It is based on instantaneous values in three-phase power systems with or without neutral wire, and is valid for steady-state or transitory operation, as well as for generic voltage and current waveforms. The p-q theory consists of an algebraic transformation (Clarke transformation) of the three-phase voltages and currents in the a-b-c coordinates to the d-q coordinates, followed by the calculation of the p-q theory instantaneous power components:
\[ \begin{bmatrix} v_0 \\ v_a \\ v_b \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & -1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix} \begin{bmatrix} v_0 \\ v_q \\ v_d \end{bmatrix} \]
\[ \begin{bmatrix} i_0 \\ i_a \\ i_b \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & -1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_0 \\ i_q \\ i_d \end{bmatrix} \]

\[ p_0 = v_0 \cdot i_0 \] instantaneous zero-sequence power
\[ p = v_a \cdot i_a + v_b \cdot i_b \] instantaneous real power
\[ q = v_a \cdot i_a - v_b \cdot i_b \] instantaneous imaginary power
The power components \( p \) and \( q \) are related to the same \( \alpha-\beta \) voltages and currents, and can be written together:

\[
\begin{bmatrix}
p \\
q
\end{bmatrix} = \begin{bmatrix}
v_\alpha & v_\beta \\
-v_\beta & v_\alpha
\end{bmatrix} \begin{bmatrix}
i_\alpha \\
i_\beta
\end{bmatrix}
\]

(14)

These quantities are illustrated in Fig 6 for an electrical system represented in \( a-b-c \) coordinates and have the following physical meaning:

\[
\overline{p}_0 = \text{mean value of the instantaneous zero-sequence power} \quad \text{– it means the energy per time unity that is transferred from the power supply to the load through the zero-sequence components of voltage and current.}
\]

\[
\overline{p} = \text{alternated value of the instantaneous zero-sequence power} \quad \text{– it means the energy per time unity that is exchanged between the power supply and the load through the zero-sequence components. The zero-sequence power only exists in three-phase systems with neutral wire. Furthermore, the systems must have unbalanced voltages and currents and/or 3rd harmonics in both voltage and current of at least one phase.}
\]

\[
\overline{p} = \text{mean value of the instantaneous real power} \quad \text{– corresponds to the energy per time unity which is transferred from the power supply to the load, through the \( a-b-c \) coordinates, in a balanced way (it is the desired power component).}
\]

\[
\overline{q} = \text{instantaneous imaginary power} \quad \text{– corresponds to the power that is exchanged between the phases of the load. This component does not imply any transference or exchange of energy between the power supply and the load, but is responsible for the existence of undesirable currents, which circulate between the system phases. In the case of a balanced sinusoidal voltage supply and a balanced load, with or without harmonics, \( \overline{q} \) (the mean value of the instantaneous imaginary power) is equal to the conventional reactive power (\( \overline{q} = 3 \cdot V \cdot I_1 \cdot \sin \phi \)).}
\]

It is also possible to conclude from Fig. 7 that the active filter capacitor is only necessary to compensate \( \overline{p} \) and \( \overline{p}_0 \), since these quantities must be stored in this component at one moment to be later delivered to the load. The instantaneous imaginary power \( q \), which includes the conventional reactive power, is compensated without the contribution of the capacitor. This means that, the size of the capacitor does not depend on the amount of reactive power to be compensated.

To calculate the reference compensation currents in the \( \alpha-\beta \) coordinates, (14) is inverted, and the powers to be compensated \( \overline{p} - \overline{p}_0 \) and \( q \) are used:

\[
\begin{bmatrix}
i_{\alpha}^* \\
i_{\beta}^*
\end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix}
v_\alpha & -v_\beta \\
v_\beta & v_\alpha
\end{bmatrix} \begin{bmatrix}
\overline{p} - \overline{p}_0 \\
q
\end{bmatrix}
\]

(15)

Since the zero-sequence current must be compensated, the reference compensation current in the \( \theta \) coordinate is \( i_0 \) itself:

\[
i_{\theta}^* = i_0
\]

(16)

In order to obtain the reference compensation currents in the \( a-b-c \) coordinates the inverse of the transformation given in (10) is applied:

\[
\begin{bmatrix}
i_{\alpha}^* \\
i_{\beta}^* \\
i_{\gamma}^*
\end{bmatrix} = \sqrt{3} \begin{bmatrix}
1/\sqrt{2} & 1/\sqrt{2} & 0 \\
1/\sqrt{2} & -1/2 & \sqrt{3}/2 \\
1/\sqrt{2} & -1/2 & -\sqrt{3}/2
\end{bmatrix} \begin{bmatrix}
i_{\theta}^* \\
i_{\alpha}^* \\
i_{\beta}^*
\end{bmatrix}
\]

(17)

The calculations of the p-q theory are synthesized in Fig 8 and correspond to a shunt active filter control strategy for constant instantaneous supply power. This approach,
when applied to a three-phase system with balanced sinusoidal voltages, produces the following results:
- The phase supply currents become sinusoidal, balanced, and in phase with the voltages. (in other words, the power supply “sees” the load as a purely resistive symmetrical load);
- The neutral current is made equal to zero (even 3rd order current harmonics are compensated);
- The total instantaneous power supplied, \( p_{3s}(t) = v_a \cdot i_a + v_b \cdot i_b + v_c \cdot i_c \) (18) is made constant.

\[
\text{Load Instantaneous Power: } p_{3s}(t) = v_a \cdot i_a + v_b \cdot i_b + v_c \cdot i_c
\]

\[
\text{Source Instantaneous Power: } p_{3s}(t) = v_a \cdot i_a + v_b \cdot i_b + v_c \cdot i_c
\]

The p-q theory also permits a control strategy for the shunt active filter to be used when voltages are distorted and/or unbalanced and sinusoidal supply currents are desired [13]. However with this strategy the total instantaneous power supplied will not be constant, since it is not physically possible to achieve both sinusoidal currents and constant power in a system with unbalanced and/or distorted voltages.

V. ACTIVE FILTER SIMULATION RESULTS
Figure 9, presents simulation results using Matlab/Simulink [14, 15] for a three-phase power system with a shunt active filter with control based on the p-q theory. It includes the following waveforms, corresponding to two-cycles of steady-state operation: phase voltages \((v_a, v_b, v_c)\); load phase and neutral currents \((i_a, i_b, i_c, i_n)\); total instantaneous power at load \((p_3)\) and source \((p_{3s})\); and source phase and neutral currents \((i_{sa}, i_{sb}, i_{sc}, i_{sn})\).

VI. ACTIVE FILTER EXPERIMENTAL RESULTS
The next figures illustrate the performance for different operating conditions of a shunt active filter with control based on the p-q theory. This active filter was developed at University of Minho, and uses digital control implemented with a standard microcontroller [16, 17]. Although the power system and the active filter are three-phase, for the sake of better understanding, the figures only show waveforms for phase a: reference compensation current \((i_{a*})\), compensation current \((i_a)\), phase voltage \((v_a)\), and supply current \((i_{sa})\).
In the first case (Fig. 10) the power system operates with a linear and almost “pure” L load, (the phase supply current, \(i_{sa}\), is almost 90° delayed regarding to the same phase voltage, \(v_a\)). The active filter controller calculates the reference compensation current (\(i_{ca\ast}\)), and as soon as its inverter is turned-on the compensation current (\(i_{ca}\)) produced by the inverter makes \(i_{sa}\) in phase with \(v_a\). In other words, the shunt active filter compensates the load power factor and it does so almost instantaneously.

The second operating condition shows the power system operating both with a non-linear load (three-phase rectifier) and a linear load (RL load). The source current is distorted, and delayed in relation to the voltage. After the active filter inverter is turned-on, \(i_{sa}\) becomes sinusoidal and in phase with \(v_a\) (Fig. 11). Once again, the compensation is immediate.

In the third case the power system operates with only a non-linear load (three-phase rectifier with RL load at DC side). After turning-on the active filter inverter \(i_{sa}\) becomes sinusoidal (Fig. 12). Figure 13 shows, for this same case, the waveforms of phase voltage and supply current separately, for operation with active filter off and on. It is possible to see that the abrupt changes in load current produce notches in the system voltage. When the active filter compensates the currents these notches disappear.

Figure 14 illustrates the response of the system to a load change. At the beginning it operates with a linear RL load and then a non-linear load (three-phase rectifier) is connected. When the active filter is on, the load changing is fully compensated in a half cycle. The current increases in amplitude because the demanded energy becomes larger with the new load, but the source current remains sinusoidal and in phase with the voltage. The figure also shows the compensation current (\(i_{ca}\)) when the active filter is on.

It is important to explain that, the ripple observed in the supply current waveform when the shunt active filter in operating occurs due to the inverter commutation. However, it only seems to be relevant because the loads used to obtain the experimental results were relatively small. Operating with larger loads the current ripple would be negligible. Besides, this is a high-frequency ripple, which is easily filtered by the power system.
VII. CONCLUSIONS

This paper deals with problems related with harmonics in power system networks. Several international standards issued to control power quality problems are briefly described and some important tools to analyse electrical circuits with non-sinusoidal waveforms are introduced and evaluated. Among other application, these tools are useful in the implementation of control algorithms for active filters.

Active filters are an up-to-date solution to power quality problems. Shunt active filters allow the compensation of current harmonics and unbalance, together with power factor correction, and can be a much better solution than the conventional approach (capacitors for power factor correction and passive filters to compensate for current harmonics).

Three different methods to control active filters are presented: the FBD Method, the Synchronous Reference Method, and the p-q Theory.

As an application example of these methods, the p-q theory was used as control scheme in the implementation of a shunt active filter. The solution shows to be simple and effective, since all the calculations are algebraic operations, which can be made through integer arithmetic. As a result, it was possible to implement the digital controller using a standard microcontroller, with minimum additional hardware.

Experimental results and simulations show that the shunt active filter presents good dynamic and steady-state response. It can perform harmonic currents compensation, together with power factor correction. It can also compensate for load current unbalances, eliminating the current in the neutral wire. Therefore, it allows the power source to see an unbalanced reactive non-linear load, as a symmetrical resistive load.

VIII. REFERENCES