GIMC'04
XV Congresso Italiano di Meccanica Computazionale
AIMETA Associazione Italiana di Meccanica Teorica e Applicata

A SIMPLE MICRO-MECHANICAL MODEL FOR THE HOMOGENISED LIMIT ANALYSIS OF MASONRY WALLS

G. MILANI[1], P. B. LOURENÇO[2], A.TRALLI[1]

[1] *Dipartimento di Ingegneria, Università di Ferrara, 44100 Ferrara*
[2] *Department of Civil Engineering, School of Engineering, University of Minho, Azurem, 4800 Guimaraes (PT)*

SOMMARIO
Nella presente memoria viene utilizzato un semplice modello micro - meccanico [1] [2] per l'analisi a collasso omogeneizzata di pareti in muratura caricate nel piano. Le superfici di crisi omogeneizzate, ottenute attraverso la risoluzione di un problema di ottimizzazione vincolata sulla cella elementare, sono implementate in codici di calcolo agli elementi finiti per l'analisi limite statica e cinematica.
Un caso di interesse tecnico viene trattato nel dettaglio con lo scopo di mostrare l'accuratezza dei risultati ottenuti.

ABSTRACT
A simple micro-mechanical model for the homogenised limit analysis of in-plane loaded masonry walls is developed with the aim to obtain the homogenised failure surfaces for masonry. The strength domains so obtained are implemented in finite element limit analysis codes and meaningful experimental tests are numerically treated both with a lower and an upper bound approach. A detailed comparison between experimental data available and numerical results obtained using the model proposed are presented for a set of shear walls. The example show the efficiency of the homogenised technique with respect both to the accuracy of the results and to the reduced number of finite elements required.

1. INTRODUCTION

In recent years, there has been a growing interest in unreinforced masonry structures, in order to provide efficient tools for better understanding their complex behaviour. Of course, we are still far from having robust and efficient computational tools, especially for static analyses of actual large scale masonry structures under increasing in-plane loads.

Two main approaches have been developed for the constitutive description of masonry material, known in the technical literature as macro modelling and micro modelling.

The first one does not make distinction between bricks and mortar and averages the effect of joints through the formulation of a fictitious continuous material. Nevertheless, it appears really difficult to take into account some distinctive aspects of masonry, such as anisotropy in the inelastic range and the post peak softening, closely related to the behaviour of its constituent materials (mortar and bricks) and to its geometry (bond pattern, thickness of joints).

The alternative micro modelling consists in representing separately mortar joints and bricks; in some cases, for joints the theory of interfaces has been applied. Nevertheless, some drawbacks of this approach, related to the necessity of modelling separately bricks and mortar and which limit its applicability to small panels, are worth noting.

Despite the wide employment of the homogenisation theory for modelling masonry structures in the elastic field, only few papers take advantage of this technique in the inelastic range. In fact, the use of f.e.m for the homogenisation in the inelastic range requires a prohibitive computational effort, since the field problem has to be solved numerically for every time step in any Gauss point. For these reasons, limit analysis combined with a simplified homogenisation technique seems to be a promising tool for design purposes. Furthermore, it requires a reduced number of material parameters and allows the study of masonry taking into account the texture only at a cell level.

In the present paper, some numerical results concerning a set of shear walls are reported. The simple micro mechanical model for the homogenised limit analysis of masonry already presented in [1] [2] by authors is here employed for the derivation of the homogenised failure surfaces, which are then implemented in lower and upper bound f.e. limit analysis codes.

2. HOMOGENISATION IN THE RIGID-PLASTIC CASE

For the important case of periodic arrangements of bricks and mortar, the homogenisation techniques can be profitable used both in the elastic and inelastic range, so taking into account the micro-structure of the brickwork only at a cell level. This leads to a significant simplification of the numerical models adopted for study entire walls, especially for the inelastic case.

Averaged quantities representing the macroscopic stress and strain tensors are introduced as follows:

$$\begin{gathered} E_{ij} = <\varepsilon_{ij}> = \frac{1}{A}\int_Y \varepsilon_{ij}(\mathbf{u})\,dY \\ \Sigma_{ij} = <\sigma_{ij}> = \frac{1}{A}\int_Y \sigma_{ij}\,dY \cdot \end{gathered} \tag{1}$$

where A stands for the area of the elementary cell, the lower letters indicate the local quantities (stresses and strains) and <*> is the average operator.

Periodicity conditions are imposed on stress and displacement fields as follows:

$$\begin{cases} \mathbf{u} = \mathbf{E}\mathbf{y} + \mathbf{u}^{per} & \mathbf{u}^{per} \text{ on } \quad \partial Y \\ \boldsymbol{\sigma}\mathbf{n} \quad \text{anti-periodic} & \text{on} \quad \partial Y \end{cases} \tag{2}$$

where $\mathbf{u}^{per}$ is a periodic displacement field, $\mathbf{E}$ represents the macroscopic strain tensor and ∂Y is the boundary of the elementary cell.

Let us indicate with S^m, S^b and $S^{\hom}$ respectively the strength domains of the mortar, of the bricks and of the homogenised macroscopic material.

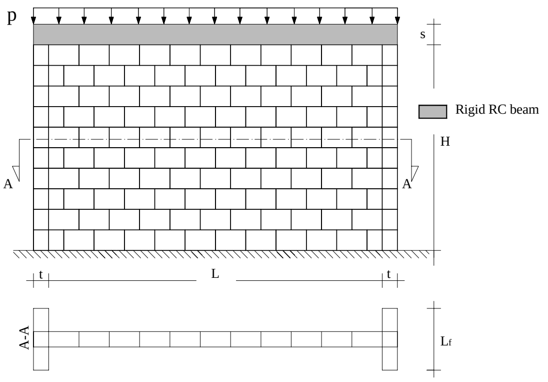

Figure 1: Geometry and loads of ETH Zurich shear wall (L= 3300 mm; t=150 mm; s=160 mm L_f=300 mm).

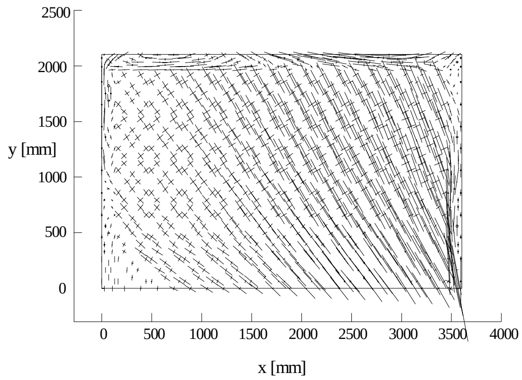

Figure 2: Principal stress distribution at collapse from the lower bound analysis

It has been shown by Suquet ([3], see also [4]) that the $S^{\hom}$ domain of the equivalent homogenised material is defined in the space of the macroscopic stresses as follows:

$$S^{\hom} = \left\{ \Sigma \mid \begin{cases} \Sigma = <\boldsymbol{\sigma}> = \dfrac{1}{A}\displaystyle\int_Y \boldsymbol{\sigma} dY & (a) \\ div\boldsymbol{\sigma} = \mathbf{0} & (b) \\ [\![\boldsymbol{\sigma}]\!]\mathbf{n}^{\text{int}} = \mathbf{0} & (c) \\ \boldsymbol{\sigma}\mathbf{n} \quad \text{anti-periodic} \quad \text{on} \quad \partial Y & (d) \\ \boldsymbol{\sigma}(\mathbf{y}) \in S^m \quad \forall \mathbf{y} \in Y^m \ ; \ \boldsymbol{\sigma}(\mathbf{y}) \in S^b \quad \forall \mathbf{y} \in Y^b & (e) \end{cases} \right\} \tag{3}$$

where $[\![\boldsymbol{\sigma}]\!]$ is the jump of micro-stresses across any discontinuity surface of normal $\mathbf{n}^{\text{int}}$. Furthermore, conditions (3-a) and (3-d) are derived from periodicity, condition (3-e) represents the material admissibility, while condition (3-b) imposes the micro-equilibrium.

3. NUMERICAL RESULTS

Several numerical simulations have been carried on by the authors in [1] and [2] in order to test the accuracy of the results obtained using the homogenised approach proposed. The comparisons presented are referred both to the elementary cell and to experimental tests on entire panels.

In this Section, an homogenised limit analysis on a set of shear walls tested by Guggisberg and Thürlimann (ETH Zurich clay brick masonry shear walls, [5]) is reported. The geometry of the walls is depicted in Figure 1. The dimension of the bricks is assumed to be 300x200x150 mm^3, whereas the thickness of joints is supposed infinitesimal. Vertical flanges have the width of a single unit and a distributed vertical load P=415 kN is applied on the rigid RC beam on the top. Both for mortar joints and units, a Mohr-Coulomb failure criterion in plane stress is adopted. It has to be emphasized that the results reported in literature provide insufficient information on the mechanical characteristics of the components for a full parametric identification of the model in plane stress. For this reason, mechanical characteristics of constituent materials are assumed to fit experimental data reported.

Experimental evidences show a very ductile response, so justifying the use of limit analysis for predicting the collapse load, with tensile and shear failure along diagonal stepped cracks.

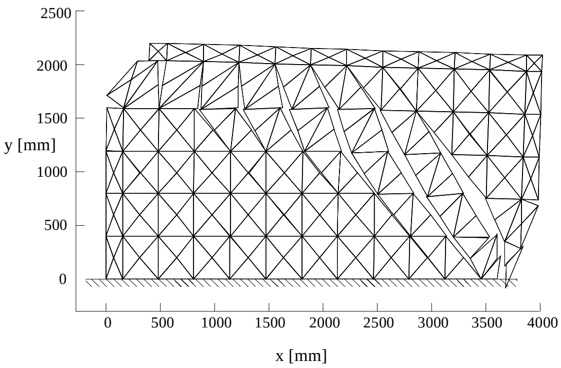

Figure 3 : Velocities at collapse from the upper bound analysis.

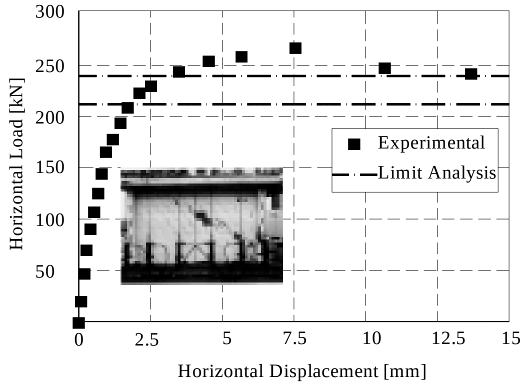

Figure 4: Comparison between experimental load-displacement diagram and the homogenised limit analysis for the ETH Zurich shear test.

In Figure 2 and Figure 3 respectively the principal stress distribution at collapse from the lower bound analysis and the velocities at collapse from the upper bound analysis are reported. Good agreement is found among the model here proposed, the incremental elastic-plastic analysis reported in [5] and experimental data.

Finally, in Figure 4 a comparison between the numerical failure loads provided respectively by the lower and upper bound approaches and the experimental load-displacement diagram is reported. Collapse loads $P^{(-)}$= 210 kN and $P^{(+)}$=245kN are numerically found using a model with 288 triangular elements, whereas the experimental failure shear at the base is approximately P=250 kN.

REFERENCES

[1] Milani G., Lourenço P. B., Tralli A.: Homogenised limit analysis of masonry walls. Part I: failure surfaces, *Submitted*, (2004).

[2] Milani G., Lourenço P. B., Tralli A.: Homogenised limit analysis of masonry walls. Part II: structural example, *Submitted*, (2004).

[3] Suquet P.: Analyse limite et et homogeneisation, *C.R. Acad. Sci., Paris*; 296 (Serie II): 1355-1358 (1983).

[4] Taliercio A., Sagramoso P.: Uniaxial strength of polymeric-matrix fibrous composites predicted through a homogenization approach, *International Journal of Solids and Structures*, 32 (14): 2095-2123 (1995).

[5] Lourenço P. B., Computational strategies for masonry structures. PhD Thesis. Delft University of Technology, the Netherlands (1996).