SECOND AND THIRD CLASS PARTICLES IN TASEP

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Abstract. We consider the nearest neighbors one-dimensional totally asymmetric simple exclusion process starting with ones to the left of the origin, a second class particle at the origin, a third class particle at site 1 and no particles to the right of site 1. We show that the probability that the third class particle is to the right of the second class particle at time t converges to $2/3$ as $t \to \infty$. We also consider the asymmetric exclusion process with transition rates having a positive mean and show that if the system starts with a product measure with densities $\lambda > \rho$ to the left and right of the origin, respectively, then the position of the second class particle at time t divided by t converges in distribution to a uniform random variable in the interval $[-(\rho - \lambda), \rho - \lambda]$, extending a result by the first author and Kipnis.

1. Introduction

The Asymmetric Simple Exclusion process is one of the most studied interacting particle systems. In this process, particles evolve on $\mathbb{Z}$ according to interacting random walks with an exclusion rule which prevents to have more than a particle per site. The dynamics is as follows. Fixed a probability $p(\cdot,\cdot)$ on $\mathbb{Z}\times\mathbb{Z}$, each particle independently of the others, waits a mean 1 exponential time, after which, being at the site x it jumps to a site y at a rate $p(x, y) := p(y - x)$. If the site is occupied the jump is suppressed in order to respect the exclusion rule and after that it restarts. Without losing generality we assume $\sum_x p(x) = 1$. This continuous time Markov process η_t has state space $\{0, 1\}^{\mathbb{Z}}$ and for a site $x \in \mathbb{Z}$, $\eta_t(x)$ denotes the quantity of particles at that site at the macroscopic time t. Then, if $\eta_t(x) = 1$ the site x is occupied otherwise it is empty.

When the transition probability rate $p(\cdot)$ has positive and finite mean $\gamma > 0$, the process is called Asymmetric Exclusion Process (AEP). The Asymmetric Simple Exclusion Process (ASEP) will be used when the jumps are nearest neighbor with $p(1) = p$ and $p(-1) = q$ with $p+q = 1$ and $p \neq 1/2$. The Totally Asymmetric Simple Exclusion process (TASEP) if in this last case $p = 1$.

Since the work of Rezakhanlou, it is know that starting this process from an initial measure associated to a profile (see [6]) it has an hydrodynamic limit given by the inviscid Burgers equation: $\partial_t u(r, t) + \gamma\nabla u(r, t)(1 - u(r, t)) = 0$.

For $\rho \in [0, 1]$ denote by ν_ρ the Bernoulli product measure of parameter ρ. It is know that ν_ρ is an invariant measure for this process and that all invariant and translation invariant measures are convex combinations of ν_α if $p(\cdot,\cdot)$ is such that $p_t(x, y)+p_t(y, x) > 0$, $\forall x, y \in \mathbb{Z}^d$ and $\sum_x p(x, y) = 1$, $\forall y \in \mathbb{Z}^d$, see [4].

A *second class particle* is a particle that behaves with holes as a particle and with particles as a hole: if there is a second class particle at site x, then it jumps to y with rate $p(y - x)$ if y is empty and interchanges positions with a particle at y at rate $p(x - y)$.

Let $\nu_{\lambda,\rho}$ be a product measure with density λ to the left of the origin and ρ to the right of it. Ferrari and Kipnis [1] start the TASEP with a configuration chosen accordingly to $\nu_{\lambda,\rho}$ with $0 \le \rho < \lambda \le 1$ put a second class particle at site 0 and call X_t its position at time t. Then they prove that X_{tN}/N converges as $N \to +\infty$ to a Uniform random variable with support on $[-t, t]$. The extension of last result to the case of a AEP is straightforward, but for completeness we state this result in this general case and we make a sketch of its proof, following the same arguments as [1]. Almost sure convergence has been proven by Mountford and Guiol [5], Ferrari and Pimentel [3] and Ferrari, Martin and Pimentel [2].

We consider the process with different classes of particles. Holes can be considered as particles of class ∞. A class-m particle at x interchanges positions with class-k particle at site y at rate

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$p(y-x)$ if $m<k$ and at rate $p(x-y)$ if $m>k$. That is, a pair of class-m and class-k particles behaves as particle-hole if $m<k$ and as hole-particle if $m>k$; particles of the same class interact by exclusion. For example if a second class particle attempts to jump to a site occupied by a first class particle, the jump is suppressed but if instead it attempts to jump to a site occupied by a third class particle, then they exchange positions. As a consequence, the higher the degree of the class of a particle the less is its priority.

We start the process from a deterministic configuration, denoted by ξ, that has all negative sites occupied by (first class) particles, the origin and site 1 are occupied by a second class and a third class particle, respectively, and all sites to the right of site 1 are empty. We show that for the TASEP from the configuration ξ, the probability of the second class particle jumping to the right of the third class particle at time t converges to $2/3$ as $t\to\infty$. The same argument shows that the limiting value equals $(p+1)/3p$ for the ASEP.

References

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