A Nonscale Growth Model with R&D and Human Capital Accumulation

Maria-Joao Ribeiro Thompson*
University of Warwick / University of Minho
c/o Universidade do Minho
Escola de Economia e Gestao
Gualtar, Braga
Portugal
E-mail:mjribeiro@eeg.uminho.pt

Abstract

This paper presents an endogenous growth model that includes research and development and human capital accumulation.

The model's specification builds on the R&D-based structure of Romer's [1990] model and introduces two functions: (1) A specification for the production of new designs that assumes no externalities and no inventions before time zero; and (2) A specification for the accumulation of human capital technically similar to that in Lucas [1988].

The model displays two main results. The first is that it eliminates the scale-effects prediction which is common to most R&D-based growth models, but which is not empirically supported.

Secondly, the model offers a new prediction that growth depends positively on the ratio of final-good workers to researchers. Thus the model provides a theoretical explanation as to why developed countries have had rising numbers of researchers but not rising growth rates in the twentieth century.

JEL Classification: O0; O3; O4; D5.
Keywords: endogenous growth; research and development; human capital accumulation; scale-effects prediction; final-good workers to researchers ratio.

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1 Introduction

In the majority of endogenous growth literature the respective roles of research and development (R&D) and human capital accumulation in generating sustained positive growth have been studied separately from each other.

In this paper, we develop a model that integrates creation of designs and human capital accumulation into a single framework. This growth model has an R&D-based structure in which the engine of growth is human capital accumulation.

The model displays two main results. The first is that it eliminates the scale-effects prediction that growth depends positively on the number of researchers and, consequently, on the size of the economy. This scale-effects prediction is common to most R&D-based growth models but is not empirically supported.

Secondly, the model offers a new prediction that growth depends positively on the ratio of final-good workers to researchers. Thus the model can be used to provide a theoretical explanation as to why developed countries have had rising numbers of researchers but not rising growth rates in the twentieth century. The model predicts that a rise in the number of researchers will lower the growth rate unless this rise is accompanied by a proportionately equal or larger increase in the number of final-good workers.

As first identified by Jones [1995], the first-generation of R&D-based endogenous growth models, including Romer [1990], Grossman and Helpman [1991] and Aghion and Howitt [1992], fail empirically because they display the above mentioned scale-effects prediction. In fact, the labour force in developed countries has increased substantially over the last century whilst growth rates have been relatively constant or have even declined. Similarly, a positive relationship between the number of researchers and the growth rate has been empirically rejected.

Beginning with Jones [1995], many growth economists have been actively attempting to eliminate the scale-effects prediction from R&D-based models. As defined by Jones [1999], these new growth models fall into two groups. In the first group of nonscale[1] growth models lie the models of Jones [1995], Kortum [1997] and Segerstrom [1998], all of which obtain the result that the growth rate of output per-capita is proportional to the growth rate of the population, rather than the absolute population size. The growth rate of the population is assumed exogenous, which means that these models result in the neoclassical growth model's prediction that neither economic policies nor tastes impact on the economic growth rate. Moreover, in the absence of population growth, exponential economic growth cannot be sustained in this kind of models.

The second line of this latest research on scale and growth includes the works of Aghion and Howitt [1998 Chp.12], Dinoupolous and Thompson [1998], Peretto [1998] and Young [1998]. These models eliminate the scale-effects prediction by assuming that an increase in scale increases the number of products available,

[1] A nonscale growth model is a model without the scale-effects property.

leaving the amount of research effort per sector (and consequently growth) unaltered. In these models, changes in policy affect the long-run growth rate and, in addition, the models result in exponential growth in the absence of population growth.

The model we propose in this paper is a nonscale growth model which does not fall into the two groups of models referred to above.

The model is an R&D-based growth model that considers human capital accumulation.

Human capital is defined here as an individual's capacity to observe, comprehend and act accordingly upon his/her environment. Accumulation of this capacity is done by the whole educated population; either by taking extra courses, through self instruction, or learning with their family or peers.

The model's specification builds on the R&D-based structure of Romer's [1990] model and introduces two functions: (1) A specification for the production of designs that assumes no externalities and no inventions before time zero; and (2) A specification for the accumulation of human capital, technically similar to that in Lucas [1988].

The model results in multiple balanced growth paths, parameterised by the ratio of final-good workers to researchers. These parameterised solutions allow for exponential economic growth in the absence of population growth.

The scale-effects prediction is eliminated in the new model because technological progress does not depend on the number of researchers, but instead on the rate of growth of human capital.

The proposed model also carries a new result that growth depends positively on the ratio of final-good workers to researchers. That is, it predicts that in order to grow faster, a country must increase its ratio of final-good workers to researchers.

As Temple [1999] writes, the latest empirical research shows strong evidence of differences in levels and/or rates of growth across countries. This suggests that both policy and institutions affect economic growth[2], and thus points to endogenous growth models as being better than the neoclassical model at explaining reality.

In this sense, the growth model developed in this paper can be used to analyse the effects of economic policy and of international trade on the growth rate. It is found that subsidies to the research sector, or international trade of capital goods, lead to a higher long-run per-capita growth rate.

The paper is organised as follows. After this Introduction, Section 2 includes the specification and results of the new model. Section 3 analyses the model's implications regarding the elimination of the scale effects prediction, with a comparison between Jones' [1995] model and this model. This Section further

[2] Mills and Crafts [1999], too, show that the recent OECD experience shows no tendency for the equalisation of long-run growth rates.

analyses the new model's implications in terms of economic policy, trade and welfare properties. Section 4 closes the present study with Concluding Remarks.

2 Specification and Results of the Model

The preference structure adopted for this model is the standard optimising one. Infinitely lived homogeneous consumers maximise, subject to a budget constraint, the discounted value of their representative utility:

$$Max \int_0^\infty e^{-\rho t} U(C_t)dt \qquad , \qquad U(C_t) = \frac{C_t^{1-\sigma}}{1-\sigma},$$

where variable C_t is consumption in period t, ρ is the rate of time preference and $\frac{1}{\sigma}$ is the elasticity of substitution between consumption at two periods of time. A consumer facing a constant interest rate r, chooses to have consumption growing at the constant rate g_c given by the familiar Euler equation:

$$g_c = \frac{\dot{C}}{C} = \frac{1}{\sigma}(r - \rho) \tag{1}$$

Equation 1 expresses a positive relationship between the interest rate and the growth rate. In the (r, g) space this relationship is represented by an upward sloping curve that unites pairs (r, g) which constitute balanced growth paths determined by the savings decisions of households. It is called the Preferences curve[3].

As in Romer [1990], the production side can be understood as having three sectors. The final goods sector, the capital goods sector and the R&D sector. The specification of the production technology for the final goods sector picks up the production function used by Romer [1990], in the version presented by Jones [1995] and Aghion and Howitt [1998], and adapts it so as to replace labour with effective labour, in a Lucas [1988] fashion.

Thus, the final good Y is produced using, as inputs, effective labour devoted to final output L_Y^e and a number A of differentiated durable capital goods i each produced in quantity $x(i)$. All capital goods have additively separable effects on output. The production function is, then:

$$Y_t = (u_t h_t L_Y)^{1-\alpha} \int_0^{A_t} x_t(i)^\alpha di, \tag{2}$$

where, as workers are assumed to be identical, $L_Y^e = uhL_Y$ is the effective contribution of labour to final goods production. Variable u represents time devoted to working and variable h stands for a worker's level of human capital ranging from zero to infinity.

A worker with human capital level h, and endowed with one unit of time per unit of time, devotes the fraction u of his non-leisure time to current production,

[3] After Rivera-Batiz and Romer [1991].

and the remaining $1-u$ to human capital accumulation. The model implicitly assumes that the amount of leisure is fixed exogenously.

Physical capital accumulation is given by:

$$\dot{K}_t = Y_t - C_t,$$

and assuming that it takes one unit of foregone consumption to produce one unit of any type of capital good, K is related to the capital goods by the rule:

$$K_t = \int_0^{A_t} x_t(i)di,$$

People can choose whether to work in the final goods sector or in the R&D sector:

$$\overline{L} = L_Y + L_A, \tag{3}$$

where total population L is assumed to be constant.

In the R&D sector, the production function of designs that we introduce is:

$$A_t = \varepsilon(u_t h_t L_A) \qquad , \qquad 0 \leq \varepsilon < 1 \tag{4}$$

This specification involves the definition of variable A as a flow variable. We interpret research activities as follows: In period t, researchers invent A_t designs, which they sell to capital good producers. But, in the following period, the use of these patents requires researchers to reread the manual of each patent and teach the capital good producers how to use them. That is, the manuals of each patent are of no use per se, as they need to be read, each period, by someone who understands them - the researchers. This idea finds agreement with Langlois' [2001] view that much knowledge in the economy is tacit and not easily transmitted.

Equation 4 also implies that there is no exogenous discovery before time zero. That is, without R&D activities, the number of designs is zero. In other words, no invention has fallen from the sky at the time this economy is born.

It also means that a positive level of output requires a positive number of researchers, and that the higher the number of researchers L_A in one economy, the higher is the number of designs A.

Another important aspect of this specification is that it assumes that the productivity of researchers is independent from the number of designs. That is, in this model there are no positive externalities across time in the R&D process.

The zero external returns assumption can be supported with Romer's [1990] idea that whether there are increasing or decreasing returns to R&D is a philosophical question. The assumption can also be supported by Jones' [1995] argument that if on the one hand some discoveries like calculus are most likely to increase the productivity of the following researchers, on the other hand it is also likely that the most obvious ideas are discovered first, making it more difficult for the following researchers to discover new ideas.

According to equation 4, in a balanced growth path A will be growing at the same rate as the population's human capital h, as L_A is assumed constant, and u must be constant in a balanced growth path, as will be seen later:

$$\begin{aligned} A_t &= \varepsilon(u_t h_t L_A) \\ &\Rightarrow \\ g_A &= g_h \end{aligned}$$

Technological progress is dependent on human capital accumulation. Hence, we must now specify the human capital accumulation process.

The population is assumed to be skilled, and each person can increase his/her human capital by taking extra courses[4], learning by themselves, or learning with their peers and family. The human capital accumulation equation assumes a Uzawa's [1965] or Lucas' [1988] form:

$$\dot{h}_t = h_t \gamma (1 - u_t), \tag{5}$$

where, as written before, $1 - u$ is the amount of time dedicated to accumulating human capital, and γ is a constant reflecting the efficiency with which an individual's time spent absorbing new information translates into his/her accumulation of human capital.

Equation 5 says that a balanced growth path, that is a solution with a constant growth rate of h, requires a constant u. This means that, in a balanced growth path, infinitely lived people will dedicate, in each period, a constant amount of time to working and a constant amount of time to learning. Also, human capital will grow by the same constant proportion each period[5]. This result is driven by the idea that there is always something new to learn. Skilled individuals (intentionally) keep on absorbing new information, attending training activities - in short, accumulating human capital. This human capital benefits the productivity of workers in whichever sector they choose to work[6].

As mentioned above, all labour in this economy is assumed to be skilled. We have not included unskilled labour in this model as we take up Cohen's [1998] view that unskilled individuals are bound to be excluded from a developed economy. In his 1998 book, Cohen states that a worker who does not engage in the task-upgrading efforts of society as a whole is left behind.

Moving on, final good producers rent each capital good according to the profit maximisation rule:

$$\frac{dY_t}{dx_t(i)} = R_t(i),$$

[4] Assumed to be paid for by the government.

[5] If we prefer to think in terms of finitely lived individuals and infinitely lived familes, we have to assume that altruistic parents leave everything to their children, including their knowledge. This is not difficult to accept if we observe that the better educated the parents are, the greater the level of knowledge showed by their children from as early as birth.

[6] Notice that the accumulation of knowledge cannot be assumed to happen through learning-by-doing, as it would imply two different kinds of knowledge. In this model we assume that there is only a common base of knowledge which is used by workers in whatever sector they are employed.

which gives the inverse demand curve faced by each capital good producer:

$$R_t(i) = \alpha(uh_tL_Y)^{1-\alpha}x_t(i)^{\alpha-1} \tag{6}$$

Faced with given values of L_Y and r, the monopolistic capital good producer that has bought a design and owns the patent on it, will maximise its revenue minus its variable cost at every date:

$$Max \qquad \pi_t(i) = R_t(i)x_t(i) - r_tx_t(i)$$

With a constant marginal cost and a constant elasticity demand curve, this monopolistic competitor solves his profit maximisation problem by charging a monopoly price which is a markup over marginal cost:

$$\frac{d\pi_t(i)}{dx_t(i)} = 0 \Leftrightarrow R_t(i) = \frac{r_t}{\alpha}$$

The decision to produce a new capital good depends on a comparison between the discounted stream of net revenues that the patent on this good will bring in the future, and the cost P_A of the initial investment in its respective design. We assume that researchers charge no extra price for rereading, each period, the manual of each design.

The market for designs is competitive, so at every date t the total cost of each design will be equalised to the present value of the future revenues that a monopolist can extract. This means that capital good producers earn total profits of zero present value. The dynamic zero-profit/free-entry condition is then:

$$\begin{aligned} P_{At} &= \int_t^{\infty} e^{-r(\tau-t)}\pi_\tau d\tau \\ &\Leftrightarrow \\ \dot{P}_{At} &= rP_{At} - \pi_t, \end{aligned} \tag{7}$$

where the second equation is obtained assuming that there are no bubbles.

The model is solved for its balanced growth path, i.e. the equilibrium in which the variables h, A, K, C and Y grow at constant exponential rates:

Equation 1 tell us that in a balanced growth path the interest rate must be constant. Therefore $R(i)$ is also constant. Hence, having in consideration the fact that the symmetry of the model implies that $R(i) = \overline{R} = R$ and $x(i) = \overline{x} = x$, the demand function faced by each capital good producer is rewritten as:

$$x_t = (uh_tL_Y)\left[\frac{\alpha^2}{r}\right]^{\frac{1}{1-\alpha}} \tag{8}$$

Time-differentiation of equation 8 shows that in a balanced growth path, x is growing at the same rate as human capital:

$$\frac{\dot{x}}{x} = \frac{\dot{h}uL_Y\left[\frac{\alpha^2}{r}\right]^{\frac{1}{1-\alpha}}}{uhL_Y\left[\frac{\alpha^2}{r}\right]^{\frac{1}{1-\alpha}}} = \frac{\dot{h}}{h} \tag{9}$$

Let us move now to the equilibrium in the labour market. In equilibrium, the remuneration of labour will be equal in the final good and the R&D sectors. In the final good sector the wage per unit of time is equal to labour's marginal productivity:

$$w_{Yt} = \frac{dY_t}{dL_Y} = (1-\alpha)uh_t(uh_tL_Y)^{-\alpha}A_tx_t^{\alpha},$$

and in the research sector, labour's remuneration is:

$$w_{At} = \frac{dA_t}{dL_A}P_{At} = \varepsilon uh_tP_{At}$$

Equality of w_Y and w_A implies that:

$$P_{At} = \frac{1-\alpha}{\varepsilon}(uh_tL_Y)^{-\alpha}A_tx_t^{\alpha} \tag{10}$$

Log-differentiation of equation 10 shows that in a balanced growth path, P_A is growing at the rate:

$$g_{PA} = -\alpha g_h + g_A + \alpha g_x = g_h, \tag{11}$$

therefore the zero-profit condition 7 becomes:

$$g_h = \left[r - \frac{\pi}{P_A}\right] \tag{12}$$

Now, recalling the markup rule and equation 6, profits π are rewritten as:

$$\begin{aligned} \pi &= Rx - rx \\ &= (1-\alpha)\alpha(uhL_Y)^{1-\alpha}x^{\alpha} \end{aligned}$$

Hence, replacing π by its equivalent expression given above, and replacing P_A by its equivalent expression given by 10, we have:

$$\frac{\pi}{P_A} = \frac{\varepsilon\alpha uhL_Y}{A} = \alpha\frac{L_Y}{L_A}$$

Equation 12 can then be rewritten as:

$$g_h = \left[r - \alpha\frac{L_Y}{L_A}\right] \tag{13}$$

Now, total physical capital K grows at the rate:

$$\begin{aligned} K &= Ax \\ &\Rightarrow \\ g_k &= g_A + g_h = 2g_h, \end{aligned}$$

where we replace g_K with g_k, once per-capita variables grow at the same rate as aggregate variables, due to the constancy of total population.

Output per-capita grows at the same rate as capital, as can be deduced from log-differentiation of the production function:

$$\begin{aligned} Y &= (uhL_Y)^{1-\alpha} A x^{\alpha} \\ &\Rightarrow \\ g_y &= (1-\alpha) g_h + g_A + \alpha g_x = 2g_h \end{aligned}$$

Finally, the physical capital accumulation equation:

$$\dot{K} = Y - C$$

guarantees that consumption per-capita is growing at the same rate as output and capital per-capita:

$$g = g_c = g_y = g_k = 2g_h$$

Thus equation 13 implies that:

$$g = 2\left[r - \alpha \frac{L_Y}{L_A}\right] \tag{14}$$

Equation 14 links pairs (r, g) that constitute balanced growth paths resulting from the equilibrium conditions on the production side of the economy. It expresses a positive relationship between the interest rate and the growth rate and it is called the Technology curve[7]. It is upward sloping in the space (r, g).

The equilibrium balanced growth rate for this economy is found by solving the system of three equations, 1, 14, and 3 in three unknowns r, g and $\frac{L_Y}{L_A}$:

$$\begin{cases} g = \frac{1}{\sigma}(r - \rho) \\ g = 2\left(r - \alpha \frac{L_Y}{L_A}\right) \\ \overline{L} = L_Y + L_A \end{cases} \tag{15}$$

There are many balanced growth path solutions to this system[8], as the labour market equation turns out to be redundant. Let us see why.

First, we deduce the demand functions for L_Y and L_A:

$$\begin{aligned} w_Y &= \frac{dY}{dL_Y} = (1-\alpha) uh (uhL_Y)^{-\alpha} A x^{\alpha} \\ &= (1-\alpha) uhA \left[\frac{\alpha^2}{r}\right]^{\frac{\alpha}{1-\alpha}} \end{aligned}$$

[7] After Rivera-Batiz and Romer [1991].

[8] Provided parameter values are chosen so as to ensure that the growth rate is not greater than the interest rate, as, otherwise, present values will not be finite and the integral that defines consumers utility diverges.

We have lost L_Y, in the equation above. So we will try to obtain it through $\overline{L} - L_A$, after determining L_A. So:

$$w_A = \frac{dA}{dL_A} P_A = \varepsilon uh P_A$$

We know that in equilibrium:

$$w_Y = w_A = w$$

So, after replacing A with $\varepsilon(uhL_A)$, we obtain:

$$\begin{aligned} w &= (1-\alpha)uh\varepsilon(uhL_A)\left[\frac{\alpha^2}{r}\right]^{\frac{\alpha}{1-\alpha}} \qquad (16) \\ &\Leftrightarrow \\ L_A &= \frac{P_A}{(1-\alpha)uh\left[\frac{\alpha^2}{r}\right]^{\frac{\alpha}{1-\alpha}}} \end{aligned}$$

Now, we pick up equation 10 and replace x by its equivalent given by equation 8:

$$\begin{aligned} P_A &= \frac{1-\alpha}{\varepsilon}(uhL_Y)^{-\alpha} A x^{\alpha} \\ &= (1-\alpha)uhL_A\left[\frac{\alpha^2}{r}\right]^{\frac{\alpha}{1-\alpha}} \end{aligned}$$

This implies that equation 16 becomes:

$$L_A = \frac{(1-\alpha)uhL_A\left[\frac{\alpha^2}{r}\right]^{\frac{\alpha}{1-\alpha}}}{(1-\alpha)uh\left[\frac{\alpha^2}{r}\right]^{\frac{\alpha}{1-\alpha}}} = L_A$$

That is, L_A and L_Y are not determined by the model, which means that there are multiple balanced growth path solutions, one for each given value of $\frac{L_Y}{L_A}$.

Each balanced growth path is obtained through the parameterisation of the model. This means that for each given value of $\frac{L_Y}{L_A}$ there is one balanced growth path solution, characterised by a pair of r and g. This solution is represented in the space (r, g) as the point where the Technology curve and the Preferences curve cross.

The parameterised equilibrium growth rate is then obtained by solving the system:

$$\left\{ \begin{array}{c} g = \frac{1}{\sigma}(r-\rho) \\ g = 2\left(r - \alpha\frac{L_Y}{L_A}\right) \end{array} \right. \Leftrightarrow g = \frac{2}{1-2\sigma}\left[\rho - \alpha\frac{L_Y}{L_A}\right]$$

which is equivalent to:

$$g = \frac{2}{2\sigma - 1}\left[\alpha\frac{L_Y}{L_A} - \rho\right] \qquad (17)$$

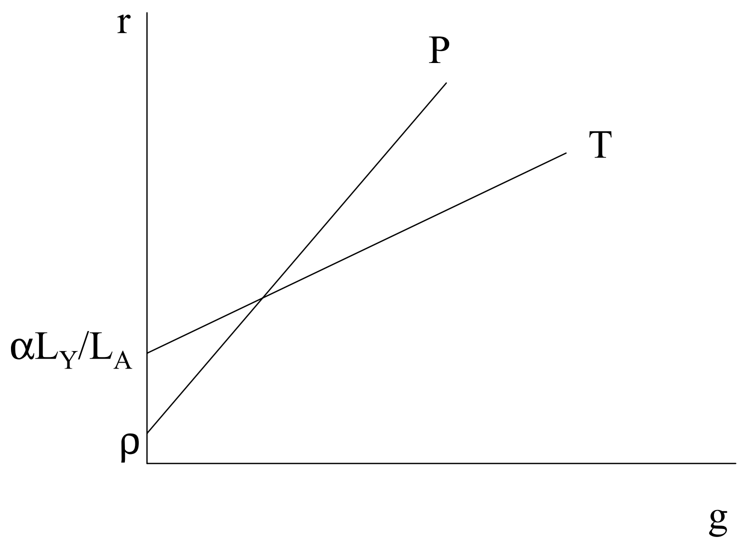

Figure 1:

Whether the equilibrium growth rate varies positively or negatively with the ratio $\frac{L_Y}{L_A}$ depends on the value of σ being greater than or smaller than 0.5, respectively. The subsequent analyses about policy and trade effects on growth are dependent on this value.

The restriction $\sigma > 0.5$ is imposed here, having in consideration that Blanchard and Fisher [1989] write that $1/\sigma$ has been observed to be normally below or close to unity. Also Barro and Sala-i-Martin [1995] use values of $\sigma = 2$ or 3 in their empirical studies on growth.

The model is illustrated in Figure 1. Under the imposed restriction on σ, the Preferences curve is steeper than the Technology curve[9].

In the Appendix, we provide a partial characterisation of the dynamics of our model around the parameterised steady-state, guided by the analyses that Barro and Sala-i-Martin [1995, Chp. 5] provided on Lucas' [1988] model, and that Arnold [2000] provided on Romer's [1990] model. We show that the equilibrium of the model can be characterised in terms of a system of five differential equations in five variables, $\chi = \frac{C}{K}$, $Z = \frac{Y}{K}$, $W = \frac{h}{A}$, L_Y and u. The steady-state of this system corresponds to the balanced growth path of our model. Then, we assume that $\sigma > \alpha^2$ and show, through phase diagram analysis, that the system is saddle-path stable.

Our model delivers the result that per-capita output growth depends positively on the ratio $\frac{L_Y}{L_A}$, meaning that the higher the proportion of workers

[9] Note that this restriction implies that $\alpha\frac{L_Y}{L_A}$ is greater than ρ.

engaged in final good production relative to those dedicated to research, the higher the growth rate of the economy.

Although surprising, this result might explain why, in many advanced countries, average growth rates have not risen or have even declined, despite an increase in their R&D intensity. Notice however, that this economy needs researchers. The solution would be indeterminate if L_A were zero.

3 Implications of the Model

3.1 No scale-effects prediction

Equation 17 shows us that the economic growth rate does not depend on the size of the population. Indeed doubling L would lead to a doubling of both L_Y and L_A, given the constant share of each in total labour in a balanced growth path. Thus doubling L would leave the growth rate unaltered.

The elimination of the scale-effects prediction is an important result. The scale-effects prediction, which characterises Romer's [1990] model and the majority of the first-generation R&D-based models is not empirically supported, as highlighted by Jones [1995].

The source of scale-effects in the first-generation of R&D-based models lies in their specification of the R&D equation. In Romer [1990] the relevant equation is:

$$\dot{A}_t = \delta A_t L_A \tag{18}$$

With such a specification, the growth rate of designs is $g_A = \delta L_A$. Therefore a balanced growth path solution, that is, a solution with a constant growth rate, requires a constant L_A, and so the growth rate is proportional to the number of workers engaged in research. This also makes the economic growth rate proportional to the size of the economy L, given the constant share of total labour dedicated to R&D.

Jones [1995] writes that this scale-effects prediction is empirically rejected. He states that the labour force has grown immensely in the developed economies over the last 25 to 100 years, and yet average growth rates have been relatively constant or have even declined. Jones adds that evidence against the R&D equation is also compelling. In the United States, for instance, the number of workers engaged in R&D grew by more than a factor of 5 from 1950 to 1988 although the average growth rate has remained relatively constant. Jones writes that even accounting for lags associated with R&D would not reverse the rejection of the scale-effects prediction.

Jones [1995] proposes an extension to Romer's model with the purpose of preserving its R&D-based structure, whilst eliminating the prediction of the scale-effects. With this extension, Jones is the father of nonscale R&D-based growth models. He transforms the original R&D equation 18 into:

$$\dot{A}_t = \delta A_t^{\phi} L_{At}^{\lambda}, \tag{19}$$

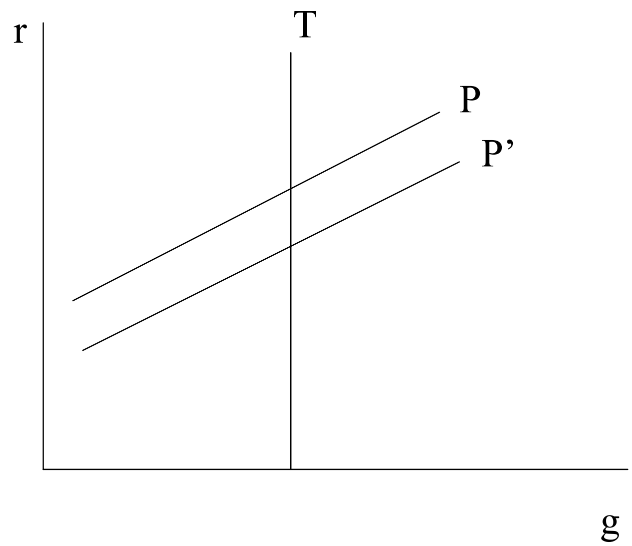

Figure 2:

where $0 < \lambda \leq 1$ and $\phi < 1$.

Equation 19 implies that the growth rate of A is:

$$\frac{\dot{A}}{A} = \frac{\delta L_A^\lambda}{A^{1-\phi}},$$

which means that a balanced growth path solution requires that:

$$g_A = \frac{\lambda g_{LA}}{1 - \phi}$$

The share of L_A in L is fixed, so $g_{LA} = g_{LY} = g_L$. The growth rate of the population is, in turn, taken as exogenous to the model.

Jones' [1995] equilibrium growth rate of output per-capita is then:

$$g_y = \frac{\lambda g_L}{1 - \phi} \tag{20}$$

In making the growth rate of output per-capita dependent on an exogenously determined variable g_L, Jones' model places the explanation for the engine of economic growth outside of the model. In doing so, Jones' model returns to being an exogenous growth model in terms of its implications for long-run growth. Hence the model does not explain whether nor how economic policies or tastes are capable of influencing the economic growth rate.

Jones' economy can be visualised with the help of Figure 2.

As the figure shows, a decrease in the value of ρ and/or σ implies a shift to the right of the Preferences curve. However, the equilibrium rate of growth remains the same.

Likewise no parameter on the production side affects the long-run economic growth.

Jones' [1995] prediction that population growth is the fundamental engine of per-capita growth can be empirically rejected. Moreover in Jones' model there is no economic growth in the absence of population growth. This is also not empirically confirmed, as Jones himself states.

Like Jones', the model introduced in this paper also preserves the structure of an R&D-based model and it does not have the scale-effects prediction. The fundamental difference between Jones' model and the model here proposed concerns the engine of growth. Jones' model is a semi-endogenous model, that is the engine of economic growth is the exogenous population growth. The model here introduced is an endogenous growth model, that is the engine of growth, which is the accumulation of human capital, is determined within the model by the existing market forces and is thus influenced by economic policies and other national characteristics. These influences are studied next.

3.2 Policy Effects on Growth

Consider a subsidy to the R&D sector financed by lump-sum taxes. This will modify the remuneration of researchers to:

$$w_A^S = \frac{dA}{dL_A} P_A = (1+s)\varepsilon u h P_A,$$

while the remuneration in the final goods sector remains the same. Therefore, the labour market equilibrium condition 10 becomes:

$$P_A{}^S = \frac{1-\alpha}{(1+s)\varepsilon}(uhL_Y)^{-\alpha} A x^{\alpha} \tag{21}$$

So:

$$\left(\frac{\pi}{P_A}\right)^S = \frac{(1+s)\alpha L_Y}{L_A}$$

which changes the Technology curve into:

$$g^S = 2\left[r - (1+s)\alpha \frac{L_Y}{L_A}\right], \tag{22}$$

meaning that the Technology curve with the subsidy lies to the left of the original one, as illustrated in Figure 3.

By enhancing the value of the ratio $\frac{L_Y}{L_A}$, the subsidy to the R&D sector positively influences the growth rate of the economy. The new equilibrium growth rate is:

$$g = \frac{2}{2\sigma - 1}\left[(1+s)\alpha \frac{L_Y}{L_A} - \rho\right] \tag{23}$$

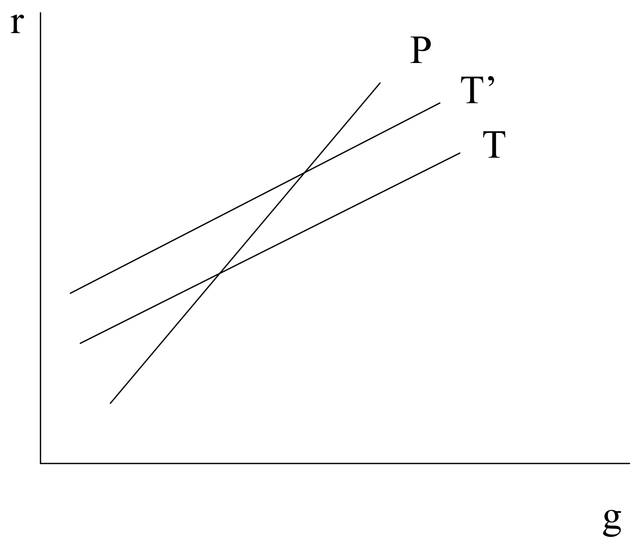

Figure 3:

The intuition of this effect is that the subsidy allows equilibrium in the labour market to occur at a lower patent price. This lower cost relative to capital good producers' revenues results in more of them wishing to enter the market. The increased demand for credit raises the interest rate, which leads to higher savings and consequently higher growth.

3.3 Trade Effects on Growth

Rivera-Batiz and Romer [1991] use Romer's [1990] model to analyse the effects of trade in capital goods on growth. A similar exercise with our model, using a two-country framework, shows that trade of capital goods increases the growth rate of the economy.

The assumptions of this exercise are:

(i) The two economies are identical, that is, $L = L^*$and $A = A^*$.

(ii) The symmetry between the two countries implies that there are no opportunities for intertemporal trade along a balanced growth path;

(iii) There is only one single final consumption good (with price equal to unity). Therefore the only trade that occurs is that of capital goods;

Assuming no redundancy in the production of new designs, trade in capital goods doubles the number of capital goods available to final good producers:

$$Y^T = (uhL_Y)^{1-\alpha} \int_0^{2A} x(i)^\alpha di \tag{24}$$

Then each capital good producer sees the demand for its good double:

$$x^T = 2(uhL_Y)\left[\frac{\alpha^2}{r}\right]^{\frac{1}{1-\alpha}} = 2x^C, \tag{25}$$

Therefore profits must also double, for a constant interest rate:

$$\pi^T = 2\pi^C,$$

where the T-nomenclature stands for the trade economy and the C-nomenclature stands for the closed economy variables.

The marginal productivity of labour in the final goods sector becomes:

$$w^T = \frac{dY^T}{dL_Y} = (1-\alpha)uh(uhL_Y)^{-\alpha}2Ax^{\alpha},$$

and researchers see the demand for their inventions double, so remuneration in the research sector becomes:

$$w^T = \varepsilon uh2P_A$$

Then equilibrium in the labour market implies:

$$P_A{}^T = P_A{}^C \tag{26}$$

Now recall the zero-profit condition 12:

$$g^C = 2\left[r - \frac{\pi^C}{P_A^C}\right]$$

With trade in capital goods, it becomes:

$$g^T = 2\left[r - 2\frac{\pi^C}{P_A^C}\right], \tag{27}$$

which means, in graphical terms, that the Technology curve with trade lies to the left of the same curve in the closed economy (again as in Figure 3), resulting in an equilibrium balanced growth path with a higher growth rate of output per-capita and a higher interest rate.

The equilibrium growth rate in the economy with trade is:

$$g^T = \frac{2}{2\sigma - 1}\left[2\alpha\frac{L_Y}{L_A} - \rho\right] \tag{28}$$

The intuition behind this result is that, facing a larger market, capital good producers have higher profits and so more of them will want to enter the market. The higher demand for credit raises the interest rate. A higher interest rate in turn makes saving more appealing, which translates into a higher growth rate.

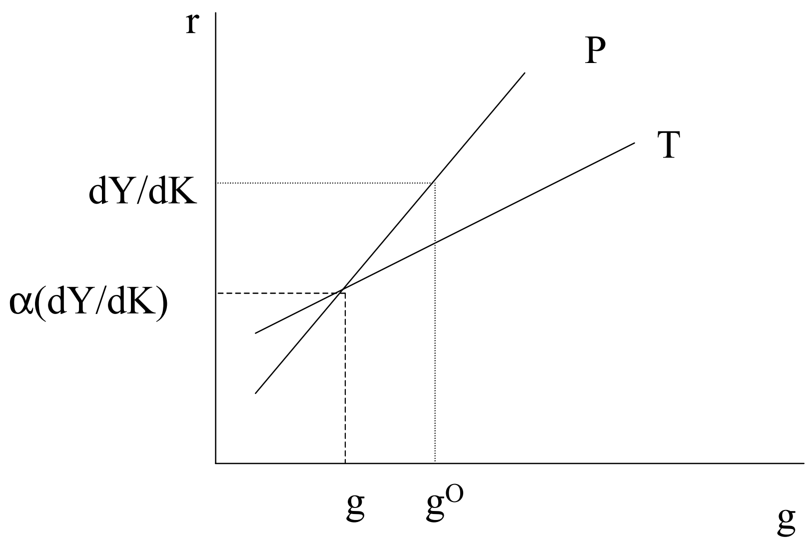

Figure 4:

3.4 Welfare Properties

Monopolistic competition in the capital goods market is responsible for a higher than marginal cost renting price of each capital good, $R = \frac{r}{\alpha}$. This implies that the equilibrium interest rate is not equal to the marginal productivity of capital, but instead it is equal to $r = \alpha R = \alpha \frac{dY}{dK}$, which is lower than the marginal productivity of capital.

Figure 4 illustrates the comparison between the model's equilibrium interest rate and the marginal productivity of capital.

The optimal solution, g^o, that is the solution with the interest rate equal to the marginal productivity of capital, corresponds to a higher point along the Preferences curve.

To achieve this equilibrium, the Technology curve would have to lie to the left of the decentralised equilibrium one. This means that the optimal equilibrium has a higher value of $\alpha \frac{L_Y}{L_A}$ than the decentralised equilibrium.

Notice that a second factor of non-optimality, besides the lower than marginal productivity remuneration of capital, is present in Romer's model but not in the model here introduced. In the new model there are no externalities from a higher stock of designs, which means that when one researcher invents one good, other researchers are not going to benefit from a higher stock of designs.

4 Concluding Remarks

In this paper, we have developed a growth model that brings human capital accumulation into an R&D framework.

In this new framework, the growth dynamics result in the ultimate engine of growth being human capital growth, which is determined by the market forces at play in the R&D monopolistic competition setting.

Human capital is interpreted as the capacity to observe, comprehend and act accordingly upon the (working) environment, influencing positively the population's productivity. Accumulation of human capital is done in a Lucas [1988] fashion by the whole population and improves not only researchers' productivity, but also the productivity of workers in the final goods sector.

In this R&D-structured model, the specification of the production of new designs assumes that there are no externalities from the existing stock of designs into the productivity of researchers. It also assumes that there are no inventions before time zero. With such specifications, technological progress does not depend on the number of researchers, but instead on the rate of human capital accumulation. As a result, the scale-effects prediction that characterises the first-generation of R&D-based models is not present in this model.

This paper aims to provide a theoretical contribution to nonscale growth theory. The model that we develop is structurally different from the previous nonscale growth models identified in the Introduction. Our model is closest in structure to Jones' [1995] model. However our model is fundamentally different from the Jones model as the engine of growth in our model is the endogenously determined human capital accumulation and not, as in the model by Jones, the exogenously determined population growth. This is important because our use of endogenously determined human capital accumulation as the engine of growth results in our model being a fully endogenous growth model. This allows us to study policy implications on growth, which is not possible in models with growth rates dependent on exogenous variables.

The model has multiple balanced growth path solutions. We have shown that unique solutions can be obtained through parameterisation of the ratio of final-good workers to researchers. We have also shown, in the Appendix that each parameterised equilibrium is saddle-path stable.

The model predicts that the growth rate of output per-capita depends positively on the ratio of final-good workers to researchers. According to this prediction, raising the number of researchers will not have a positive impact on the growth rate unless it is accompanied by a larger proportionate increase in the number of final-good workers. This might serve as a clue as to why although the developed countries have invested so much in new researchers, they have failed to experience increasing growth rates in the last century.

Appendix

In this Appendix, we provide a partial characterisation of the dynamics of our model around the parameterised steady-state.

In light of the transitional dynamics analysis that Barro and Sala-i-Martin [1995, Chp. 5] did with Lucas' [1988] model, and that Arnold [2000] did with Romer's [1990] model, we start by characterising the equilibrium of the model in terms of a system of five differential equations in five variables, $\chi = \frac{C}{K}$, $Z = \frac{Y}{K}$, $W = \frac{h}{A}$, L_Y and u.

The steady-state equilibrium is obtained by solving the system:

$$\begin{cases} \dot{\chi} = 0 \\ \dot{Z} = 0 \\ \dot{W} = 0 \\ \dot{L}_Y = 0 \\ \dot{u} = 0 \end{cases} \tag{29}$$

which is equivalent to:

$$\begin{cases} \chi = \left(1 - \frac{\alpha^2}{\sigma}\right) Z + \frac{\rho}{\sigma} \\ g_A = \alpha^2 Z - \alpha\varepsilon u L_Y W = \alpha^2 Z - \alpha\frac{L_Y}{L_A} \\ g_u = g_{LY}\frac{L_Y}{L_A} \\ \chi = (1-\alpha)Z - g_u - \gamma(1-u) + \frac{1-\alpha}{\alpha} g_A + \frac{L_Y}{L_A} \\ g_A = \gamma(1-u) - g_{LY}\frac{L_Y}{L_A} \end{cases}$$

In the steady-state, g_{LY} is zero, thus the third equation of the system says that g_u is also zero. And the fifth equation therefore states that $g_A = \gamma(1-u)$. Hence, the system becomes:

$$\begin{cases} \chi = \left(1 - \frac{\alpha^2}{\sigma}\right) Z + \frac{\rho}{\sigma} \\ \gamma(1-u) = \alpha^2 Z - \alpha\frac{L_Y}{L_A} \\ \chi = (1-\alpha)Z + \frac{1-2\alpha}{\alpha}\gamma(1-u) + \frac{L_Y}{L_A} \end{cases}$$

The steady-state value of Z is:

$$Z^* = \frac{\sigma}{\alpha^2(2\sigma-1)}\left[2\alpha\frac{L_Y}{L_A} - \frac{\rho}{\sigma}\right] = \frac{1}{\alpha^2(2\sigma-1)}\left[2\alpha\sigma\frac{L_Y}{L_A} - \rho\right]$$

Thus, the steady-state value of χ is:

$$\chi^* = \frac{2(\sigma-\alpha^2)}{\alpha(2\sigma-1)}\frac{L_Y}{L_A} - \frac{\rho(1-2\alpha^2)}{\alpha^2(2\sigma-1)}$$

The steady-state value of u is:

$$u^* = 1 + \frac{\alpha}{\gamma}\frac{L_Y}{L_A} - \frac{1}{(2\sigma-1)}\left[2\alpha\sigma\frac{L_Y}{L_A} - \rho\right]$$

And the steady-state value of W is:

$$W^* = \frac{1}{\varepsilon \left[1 + \frac{\alpha}{\gamma}\frac{L_Y}{L_A} - \frac{1}{(2\sigma-1)}\left[2\alpha\sigma\frac{L_Y}{L_A} - \rho\right]\right] L_A}$$

As analysed in this paper, all the equilibrium values depend on the parameterization of the value of $\frac{L_Y}{L_A}$.

Finally, we can deduce the steady-state growth rate of output:

$$\begin{aligned} g_y &= 2g_h = 2g_A = 2\left[\alpha^2 Z^* - \alpha\frac{L_Y}{L_A}\right] \\ &= \frac{2}{2\sigma - 1}\left[\alpha\frac{L_Y}{L_A} - \rho\right], \end{aligned} \tag{30}$$

which corresponds to the balanced growth path of this model.

Next, we analyse the transitional dynamics towards the steady-state. For that, we work with the system:

$$\begin{cases} g_\chi = \chi - \left(1 - \frac{\alpha^2}{\sigma}\right) Z - \frac{\rho}{\sigma} \\ g_z = (1-\alpha)\left[\frac{g_A}{\alpha} - \alpha Z + \frac{L_Y}{L_A}\right] \end{cases} \tag{31}$$

First, we rewrite both equations as:

$$g_z = -\alpha\,(1-\alpha)\,(Z - Z^*)$$

And:

$$g_\chi = (\chi - \chi^*) - \left(1 - \frac{\alpha^2}{\sigma}\right)(Z - Z^*)$$

which means that we have the system:

$$\begin{bmatrix} \dot{\chi} \\ \dot{Z} \end{bmatrix} = \begin{bmatrix} 1 & -\left(1 - \frac{\alpha^2}{\sigma}\right) \\ 0 & -\alpha\,(1-\alpha) \end{bmatrix} \times \begin{bmatrix} \chi \\ Z \end{bmatrix} + \begin{bmatrix} \left(1 - \frac{\alpha^2}{\sigma}\right) Z^* - \chi^* \\ \alpha\,(1-\alpha)\,Z^* \end{bmatrix}$$

So:

$$\begin{cases} \dot{\chi} = 0 \Leftrightarrow \chi = \chi^* + \left(1 - \frac{\alpha^2}{\sigma}\right)(Z - Z^*) \\ \dot{Z} = 0 \Leftrightarrow Z = Z^* \end{cases}$$

We can study the dynamics around the steady-state with a phase diagram, as in Figure 5. We assume that $1 - \frac{\alpha^2}{\sigma}$ is positive[10], which implies that $\sigma > \alpha^2$.The

[10] If this value were negative, the system would still be saddle-path stable, with a negatively sloped stable arm.

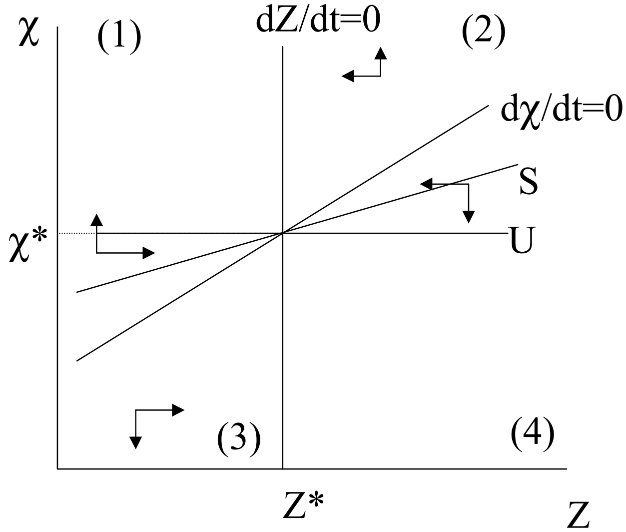

Figure 5:

stable arm is positively sloped, as can be deduced below:

$$\begin{aligned} \det \begin{bmatrix} 1-\theta & -\left(1-\frac{\alpha^2}{\sigma}\right) \\ 0 & -\alpha\left(1-\alpha\right)-\theta \end{bmatrix} &= 0 \\ &\Leftrightarrow \theta_1 = -\alpha\left(1-\alpha\right), \theta_2 = 1 \end{aligned}$$

The stable arm is the one associated with the negative eigenvalue. Let us then derive the inclination of the stable arm:

$$\begin{aligned} \begin{bmatrix} 1+\alpha\left(1-\alpha\right) & -\left(1-\frac{\alpha^2}{\sigma}\right) \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} v_{11} \\ v_{21} \end{bmatrix} &= 0 \\ &\Leftrightarrow \\ v_{11} &= \frac{\left(1-\frac{\alpha^2}{\sigma}\right)}{1+\alpha\left(1-\alpha\right)} v_{21} \end{aligned}$$

which is positive.

The unstable arm is horizontal, because:

$$\begin{aligned} \begin{bmatrix} 0 & -\left(1-\frac{\alpha^2}{\sigma}\right) \\ 0 & -\alpha\left(1-\alpha\right)-1 \end{bmatrix} \times \begin{bmatrix} v_{12} \\ v_{22} \end{bmatrix} &= 0 \\ &\Leftrightarrow \\ v_{22} &= 0 \end{aligned}$$

Concluding, if the system starts at the steady-state then it remains there. If the system starts at a point in region (1) or (4) then its dynamics takes it back to the steady-state. But if it starts somewhere in regions (2) or (3) the dynamics moves it away from the steady-state. Therefore this economy is saddle-path stable.

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