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“The Portuguese Stock Market Cycle: Chronology and Duration Dependence”

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NÚCLEO DE INVESTIGAÇÃO EM POLÍTICAS ECONÓMICAS
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Abstract

This paper tries to identify, for the first time, a chronology for the Portuguese stock market cycle and test for the presence of duration dependence in bull and bear markets. A duration dependent Markov-switching model is estimated over monthly growth rates of the Portuguese Stock Index for the period 1989-2010. Six episodes of bull/bear markets are identified during that period, as well as the presence of positive duration dependence in bear but not in bull markets.

Keywords: stock market cycles; bull and bear markets; duration dependence; Markov-switching.

JEL Classification: E32, G19, C41, C24.
1 Introduction

The identification of bull and bear markets in a stock market is a matter of huge concern for investors since an accurate prediction of the stock market trend may allow them to make significant profits or avoid painful losses. The terms ‘bull’ and ‘bear’ are commonly used to describe the state of a stock market. Chauvet and Potter (2000) define bull (bear) markets as periods of generally increasing (decreasing) market prices. In the same line, Pagan and Sossounov (2003) define them as periods of a generalized upward (downward) trend in stock prices. This definition implies that stock markets move from a bull (bear) to a bear (bull) state if prices decline (increase) for a substantial period of time since their previous peak (trough). In a more practical way, Lunde and Timmermann (2004) define them as movements in stock prices between local troughs and peaks, or vice-versa. Regarding their definitions, these authors use hazard models and extensions of Hamilton’s (1989) Markov-switching techniques to extract information from some United States (US) stock price indexes in order to identify the conditions of the respective stock market.

Markov-switching (MS) techniques were introduced in the literature by Hamilton (1989) to identify peaks and troughs in the US business cycle and the respective phases of expansion and contraction.\(^1\) This procedure models the business cycle as the outcome of a Markov process that switches between two discrete states (expansions and contractions) and regards the business cycle as an unobserved stochastic process.\(^2\) However, this approach assumes that the likelihood of switching from one state to another is not affected by its own duration. Some studies have relaxed this assumption allowing for state transition probabilities to be duration dependent. Durland and McCurdy (1994) apply such a refinement to the US real GNP growth rate series and provide evidence of duration dependence for contractions but not for expansions. A similar result is obtained by Kim and Nelson (1998) and Pelagatti (2001), but applying a Bayesian approach.\(^3\) Maheu and McCurdy (2000) extend Durland

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\(^1\) The identification of business cycle chronologies have also been under the scope of the National Bureau of Economic Research (for the US) and the Economic Cycle Research Institute (for other market oriented economies). For further details see [http://www.nber.org/cycles/cyclesmain.html](http://www.nber.org/cycles/cyclesmain.html) and [http://www.businesscycle.com/resources/cycles/](http://www.businesscycle.com/resources/cycles/).

\(^2\) Its application to the analysis of the business cycle has been quite extensive. See, among others, Krolzig (1997), Artis et al. (2004) and Schirwitz (2009).

\(^3\) Classical duration analyses have also been used to detect the presence of duration dependence in business cycles. See, among others, Sichel (1991), Diebold et al. (2003), Zuehlke (2003) and Castro (2010).
and McCurdy (1994) approach to analyse the US stock market and find evidence of duration dependence in both bull and bear markets.

The use of Bayesian techniques has some advantages over the standard (asymptotic) maximum likelihood theory for inference employed by Durland and McCurdy (1994) and Maheu and McCurdy (2000). First, Bayesian techniques do not rely on asymptotics, i.e. Bayesian inference does not depend on the sample size of the real world data. Second, inference on the latent variables is not conditional on the estimated parameters, but also incorporates the parameters’ variability. Furthermore, Pelagatti (2001, 2002) develops and employs a generalized multivariate duration dependent Markov-switching model in which the inference on the state variable is carried out using a multi-move Gibbs sampler, whilst Kim and Nelson (1998) rely on a univariate specification model and on a single-move Gibbs sampler for inference, which results in a slower convergence to the invariant distribution.

These improvements make the duration dependent Markov-switching vector autoregressive (DDMSVAR) model developed by Pelagatti (2001, 2002) an appealing model to be also employed in the study of stock market cycles. As far as we are concerned, only Chen and Shen (2007) have applied this model to investigate the presence of duration dependence in some stock markets. They show that Japan, South Korea and Hong Kong are characterised by duration dependence in bear but not in bull markets, whilst in Taiwan and Singapore the duration dependence feature holds for both bear and bull markets.

This paper extends the application of this model to the Portuguese stock market cycle, in order to identify its chronology and to test for the presence of duration dependence in the respective bull and bear markets. The few existent studies for Portugal that use MS models only look at the business cycle: Afonso et al. (2011) employ an MS model to study the changes in fiscal policy regimes and its behaviour over the economic cycle; Castro (2011) uses a DDMSVAR model to identify its chronology and find evidence of positive duration dependence in contractions but not in expansions. Thus, to our knowledge, this paper represents the first attempt of producing a similar analysis for the Portuguese stock market.

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4Other papers use this model, but to analyse the classical business cycle. See Chen and Shen (2006), Ozun and Turk (2009) and Castro (2011).

5Using a classical duration analysis, Cochran and DeFina (1995), Harman and Zuehlke (2007) and Chong et al. (2010) also find some evidence of duration dependence in the US stock market phases.
The rest of the paper is organised as follows. Section 2 presents the econometric model. The empirical results are reported and discussed in Section 3. Section 4 concludes emphasizing the main findings of this paper.

2 Econometric Model

The duration dependent Markov-switching (DDMS) model employed in this study is drawn upon the work of Pelagatti (2001, 2002, 2003). Let $y_t$ denote the growth rate of the stock market index, which is generated by the following stochastic process:

$$y_t = \mu_0 + \mu_1 S_t + \sum_{i=1}^{p} \phi_i (y_{t-i} - \mu_0 - \mu_1 S_{t-i}) + \varepsilon_t$$  \hspace{1cm} (1)

where $\varepsilon_t$ is a gaussian white noise process with mean 0 and variance $\sigma^2$, and $\phi_i$ are the coefficients of a stable autoregressive process. The term $S_t$ is a binary unobservable random variable following a Markov chain with varying transition probabilities and that takes value 1 when the stock market is bullish and 0 when it is bearish. The parameters $\mu_0$ and $\mu_0 + \mu_1$ represent, respectively, the average growth rates of $y_t$ in state 0 (bear market) and in state 1 (bull market).

Under the assumption of constant transition probability proposed by Hamilton (1989), the state variable, $S_t$, is assumed to follow a first-order Markov chain with the following transition probabilities:

$$\Pr(S_t = 0|S_{t-1} = 0) = p_{0|0} \quad \Pr(S_t = 1|S_{t-1} = 0) = 1 - p_{0|0}$$

$$\Pr(S_t = 1|S_{t-1} = 1) = p_{1|1} \quad \Pr(S_t = 0|S_{t-1} = 1) = 1 - p_{1|1}$$  \hspace{1cm} (2)

In order to achieve duration dependence for $S_t$, a Markov chain is built for the pair $(S_t; D_t)$, where $D_t$ is the duration variable. This variable ($D_t$) counts the number of periods in which $S_t$ has been in the current state. The probability of $S_t$ being in a particular state is assumed to be dependent on the previous state, $S_{t-1}$, and on the duration dependent variable, $D_{t-1}$. Hence, given $S_{t-1}, S_t$ and $D_{t-1}$, we can determine $D_t$ as follows:
\[
D_t = \begin{cases} 
D_{t-1} + 1 & \text{if } S_t = S_{t-1} \\
1 & \text{if } S_t \neq S_{t-1} 
\end{cases} 
\]  
\hspace{2cm} (3)

It is assumed that the maximum duration periods are equal to \( \tau \), with \( 0 < \tau < T \), where \( T \) is the length of the time series being modelled. This maximum value \( (\tau) \) for the duration variable \( D_t \) must be fixed, so that the Markov chain \((S_t; D_t)\) is defined on the finite state space \( \{(0,1), (1,1), (0,2), (1,2), (0,3), (1,3), \ldots, (0,\tau), (1,\tau)\} \), with a transition probabilities matrix \( P = [p_{ij}(d)] \), where each \( p_{ij}(d) = \Pr(S_t = i|S_{t-1} = j; D_{t-1} = d) \) and \( i, j = 0, 1 \).

Assuming that \( y_t \) is dependent upon the unobserved states from \( S_t \) to \( S_{t-p} \) and the duration dependent variable \( D_{t-1} \), Hamilton (1994) suggests that is always possible to write the likelihood function of \( y_t \) depending only on the state variable at time \( t \), even though in the model a \( p \)-order autoregression is present. Pelagatti (2001, 2002) notices that this can be done using the extended state variable \( S^*_t = (D_t, S_t, S_{t-1}, \ldots, S_{t-p}) \), which encompasses all the possible combinations of the states of the economy in the last \( p \) periods. He also shows that if \( \tau \geq p \), the maximum number of non-negligible states is given by \( u = \sum_{i=1}^{p}(2^i) + 2(\tau - p) \). Moreover, the transition matrix \( P^* \) of the Markov chain \( S^*_t \) will be a sparse \((u \times u)\) matrix with a maximum number of \( 2\tau \) independent non-zero elements to be estimated.

As suggested by Pelagatti (2001, 2002), a probit specification is employed to characterise the duration dependence. The use of a probit model in this framework presents two important advantages: first, it reduces to four the number \((2\tau)\) of elements in \( P^* \) to be estimated; second, it makes easier to handle in the Gibbs sampler for the Bayesian inference.

A latent variable, \( S^*_t \), can then be expressed in the following linear model:

\[
S^*_t = (\beta_1 + \beta_2 D_{t-1}) S_{t-1} + (\beta_3 + \beta_4 D_{t-1}) (1 - S_{t-1}) + \epsilon_t 
\]  
\hspace{2cm} (4)

with \( \epsilon_t \sim N(0,1) \) and the latent variable \( S^*_t \geq 0 \) when \( S_t = 1 \) and \( S^*_t < 0 \) when \( S_t = 0 \). Hence, the corresponding transition probabilities for the expansion and contraction phases, under the probit specification, can be expressed as follows: \(^7\)

\(^6\)For further details on this matrix, see Pelagatti (2001, 2002).

\(^7\)See Pelagatti (2001, 2002) and Castro (2011) for further details.
\[ p_{1|1}(d) = \Pr(S_t^* \geq 0 | S_{t-1} = 1; D_{t-1} = d) = 1 - \Phi(-\beta_1 - \beta_2 d) \]
\[ p_{0|0}(d) = \Pr(S_t^* < 0 | S_{t-1} = 0; D_{t-1} = d) = \Phi(-\beta_3 - \beta_4 d) \]

where \( d = 1, ..., \tau \), and \( \Phi(\cdot) \) is the standard normal cumulative distribution function. Now the transition probability matrix \( P^* \) is fully defined by the four parameters of \( \beta = (\beta_1, \beta_2, \beta_3, \beta_4) \). If \( \beta_2 = \beta_4 = 0 \), then we have fixed transition probabilities or duration independence.

To obtain parameter estimates from the DDMS model, we employ the Gibbs sampling approach proposed by Pelagatti (2001, 2002).\(^8\) The parameters of the model can be denoted as \( \theta = (\mu_0, \mu_1, \phi_1, ..., \phi_p, \sigma^2, \beta_1, \beta_2, \beta_3, \beta_4, \{ (S_t, D_t) \}_{t=1}^T) \), and \( \theta \) can be split into four groups:

\[
\theta = (\theta_1', \theta_2', \theta_3', \theta_4')'
\]

where \( \theta_1' = (\phi_1, ..., \phi_p, \sigma^2)' \), \( \theta_2' = (\mu_0, \mu_1)' \), \( \theta_3' = (\beta_1, \beta_2, \beta_3, \beta_4)' \), and \( \theta_4' = \{ \{ S_t, D_t \} \}_{t=1}^T \)' . Given all the observed data \( y^T = \{ y_1, y_2, ..., y_T \} \), the conditional distribution of \( \theta_k \) will be:

\[
p \left( \theta_k | y^T, \theta_{-k} \right), \quad k = 1, ..., 4.
\]

From the arbitrary initial values \( \theta^{(0)} = (\theta_1^{(0)r'}, \theta_2^{(0)r'}, \theta_3^{(0)r'}, \theta_4^{(0)r'})' \), we obtain the \( i \)th realization of \( \theta \) by considering the following procedures:

1. Draw \( \theta_1^{(i)} \) from \( p \left( \theta_1 | y^T, \theta_2^{(i-1)}, \theta_3^{(i-1)}, \theta_4^{(i-1)} \right) \)
2. Draw \( \theta_2^{(i)} \) from \( p \left( \theta_2 | y^T, \theta_1^{(i)}, \theta_3^{(i)}, \theta_4^{(i)} \right) \)
3. Draw \( \theta_3^{(i)} \) from \( p \left( \theta_3 | y^T, \theta_1^{(i)}, \theta_2^{(i)}, \theta_4^{(i)} \right) \)
4. Draw \( \theta_4^{(i)} \) from \( p \left( \theta_4 | y^T, \theta_1^{(i)}, \theta_2^{(i)}, \theta_3^{(i)} \right) \)

The \( i \)th realization of \( \theta \) is then \( \theta^{(i)} = (\theta_1^{(i)}, \theta_2^{(i)}, \theta_3^{(i)}, \theta_4^{(i)})' \). Repeating steps (1) to (4) \( I \) times, we obtain the Gibbs sequence \( (\theta^{(1)}, \theta^{(2)}, \theta^{(3)}, ..., \theta^{(I)}) \). Hence, the joint and marginal distributions of the generated \( (\theta^{(1)}, \theta^{(2)}, \theta^{(3)}, ..., \theta^{(I)}) \) will converge to the joint and marginal distribution of \( \theta = (\theta_1', \theta_2', \theta_3', \theta_4')' \) as \( I \to \infty \), i.e. \( \theta^{(I)} \xrightarrow{d} p(\theta | y^T) \).

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\(^8\)For a more detailed explanation on how the Gibbs sample is implemented, see Pelagatti (2001, 2002).

\(^9\)To perform these procedures, we use the DDMSVAR code for Ox developed by Pelagatti (2003). We thank Matteo Pelagatti for making his code available at: http://www.statistica.unimib.it/utenti/p_matteo/.
3 Empirical Results

The DDMS model is estimated over the year-on-year monthly growth rate of the Portuguese Stock Index (PSI), computed as 100 times the difference of the logarithm of the monthly PSI on a year-on-year basis \((dlPSI)\). The available monthly data for PSI were obtained from the OECD Main Economic Indicators for the period January 1988 to January 2011.\(^\text{10}\) The evolution of this index and the evolution of the respective growth rate are presented in Figure 1.

<Insert Figure 1 around here>

In particular, this figure shows that the Portuguese stock market followed the worldwide boom before the burst of the Dot Com bubble in the early 2000s. Moreover, a significant boom is also registered before the baleful consequences of the recent financial crisis has spread out to stock markets and to the world economy in general.

The estimates of the DDMS model are presented in Table 1. The scalar for the maximum duration \((\tau)\) is assumed to be 60 months (or 5 years), which shall be enough to identify the presence of duration dependence in the stock market phases. The number of lags \((p)\) was set as equal to 0. Some lags were tried, but the model did not work well, making the identification of bull and bear phases unclear.\(^\text{11}\) The priors to the vectors of parameters \(\mu\) and \(\beta\) were chosen to focus the sampling in an economically reasonable set of values.\(^\text{12}\) The Gibbs sampler was run for 11000 iterations, of which the first 1000 were discarded, and the remaining 10000 sample points were used to estimate the densities and the posteriors presented in Table 1.\(^\text{13}\)

\(^{10}\)The Portuguese Stock (or share prices) Index, with base 2005=100, includes all companies listed on the Market with Official Quotations in the Lisbon Stock Exchange. Companies included in the index are chosen in order to constitute a basket reflecting the general market climate. Monthly figures are averages of daily quotations.

\(^{11}\)The same happened when first differences of the logarithm of PSI (simple monthly growth rates) were tried. Given their high variability, it was not possible to identify bull and bear markets or the presence of duration dependence. For that reason, annual growth rates compared to the same month of the previous year are preferred.

\(^{12}\)The priors for \(\mu_0\) and \(\mu_1\) are set considering the averages of negative and positive growth rates of PSI. Other priors were tried, but results were quite similar. As in Pelagatti (2001, 2002) and Chen and Shen (2007), no specific prior was defined for \(\sigma^2\).

\(^{13}\)The means, correlograms and kernel density estimates for each parameter are reported in Appendix. All the correlograms die out before the 50th lag, hence, the choice of a burn-in sample of 1000 points seems quite reasonable for the Gibbs sampler reaching convergence to its stationary distribution.
Besides the mean and standard deviation of the posteriors, the median (50\%) and the 95\%-credibility intervals of the posterior distributions – based on the 2.5\% and the 97.5\% percentiles of the 10000 simulated draws – are also presented. Considering the mean of the posterior distributions for the estimates of $\mathbf{\mu}$, we obtain mean growth rates of about $-17.8\%$ during a bear market ($\mu_0$) and $25.7\%$ during a bull market ($\mu_0 + \mu_1$). Both estimated coefficients are statistically significant since the respective 95\%-credibility intervals do not include the value 0.

The estimates of $\mathbf{\beta}$ are displayed next. The constants ($\beta_1$ and $\beta_3$) present the expected signs and are clearly different from zero. However, the coefficients of most interest are the duration dependence coefficients $\beta_2$ and $\beta_4$. The concentration of the posterior of the parameter $\beta_2$ around zero seems to indicate that the probability of falling into a bear phase is independent of how long the stock market has been in a bull phase. This finding is reinforced by the analysis of the respective transition probabilities presented in Figure 2. There, we find that the graph for the mean of the transition probabilities from a bull to a bear market ($Pr(S_t = 0|S_{t-1} = 1, D_{t-1} = d)$) remains practically flat over time. Hence, bull markets are duration independent.

On the contrary, the posterior of $\beta_4$ is statistically away from zero since its 95\%-confidence interval does not include that value. Therefore, positive duration dependence is present in bear phases of the Portuguese stock market. Figure 2 shows that the transition probabilities of the market becoming bullish after a bear period ($Pr(S_t = 1|S_{t-1} = 0, D_{t-1} = d)$) indeed increase over time, but at a slower pace than we might suspect. Given these results, we may conclude that there is some evidence of positive duration dependence in bear markets, but no duration dependence is found for bull markets.\textsuperscript{14}

\textsuperscript{14}Identical results were obtained by Chen and Shen (2007) for Japan, South Korea and Hong Kong, employing a similar DDMS model. Using different duration dependent MS models, Maheu and McCurdy (2000) show the presence of positive duration dependence in bull and bear markets in the US. Cochran and DeFina (1995) and Harman and Zuehlke (2007) confirm those findings with a classical duration analysis, but Chong et al. (2010) only find evidence of duration dependence for the whole cycles.
Next we intend to identify the periods of bull and bear markets estimated by the model. The estimated probabilities for the Portuguese stock market being in a bull market over the period January 1989 to January 2011 are presented in Figure 3. The model proved to have a reasonable capability of discerning bull and bear markets, as the probabilities of a bull market tend to assume high and low values. These probabilities can be used to identify the turning points (peaks and troughs) in the Portuguese stock market. Making use of Hamilton’s (1989) 0.5-rule to determine the state of the economy, we end up with the stock market cycle chronology presented in Table 2.

We identify six bull and five bear market phases during the period in analysis.\textsuperscript{15} Three of the bear markets coincide reasonably well with world crises.\textsuperscript{16} The bear market in the beginning of the 1990s (August 1990 - May 1993) can be linked to some factors that affected the world economy during that period, like some prevailing repercussions of the stock crash in the late 1980s in the US, the Golf War, the German reunification, the speculative attacks to the European Exchange Rate Mechanisms and the slowdown in economic growth. The early 2000s bear market (November 2000 - August 2003) is in line with the world crisis caused by the burst of the Dot Com bubble and the September 2001 attacks in the US. The crash in the period December 2007 to September 2009 can be seen as a consequence of the financial crises that affected the world in the late 2000s. Unfortunately, the respective repercussions might not have left the markets yet since another bear market seems to have started in May 2010. Finally, the other two bear markets (December 1994 - January 1996 and February 1999 - November 1999) are more related to internal adjustments of the Portuguese stock market to the previous bull phases and to the internal economic sentiment.

Thus, the DDMS model seems to provide a reasonable picture of the Portuguese stock market cycle over the last two decades and also some support for the presence of positive duration dependence in bear markets but not in bull markets.

\textsuperscript{15}Note that, on average, bear markets tend to last the same as bull markets.

\textsuperscript{16}Not surprisingly, they also match quite well the three contractionary periods identified by Castro (2011) for the Portuguese business cycle over the period in analysis. Moreover, the bull markets are substantially related to the periods of economic expansion in Portugal.
4 Conclusions

A reasonable effort has been made in the literature to identify business cycle chronologies in some economies and the presence of duration dependence in the respective phases of expansion and contraction. However, similar studies for the stock market cycles are more scarce. In this paper, we intend to enrich the literature with an analysis of the Portuguese stock market covering two important issues: (i) the identification of a chronology for the stock market cycle; (ii) and, the study of the presence of duration dependence in the respective phases of the cycle.

A DDMS model is estimated over year-on-year monthly growth rates of the Portuguese Stock Index for the period of January 1989 to January 2011. This model proved to have a good capability of discerning bull and bear markets and in finding the presence of duration dependence. In particular, it was able to identify six episodes of bull/bear markets in the Portuguese stock market over the last two decades. The model also allowed us to detect the presence of positive duration dependence in bear markets, whilst bull markets revealed to be duration independent. Therefore, we can conclude that the likelihood of a bear market ending increases over time, but during bull markets it remains constant.

The conclusions reached in this study with the DDMS model are quite encouraging to extend the analysis to other countries and to other areas where cycles might be present, like in housing markets. We believe that a deeper analysis of stock and housing markets cycles, and respective phases, may provide useful information to better understand the economic business cycles and the presence of duration dependence in their phases.

References


List of Tables and Figures

Table 1 - Estimates of the DDMS model (January 1989 - January 2011)

| Parameter | Prior mean | var. | | Posterior mean | st.dev. | 2.5% | 50.0% | 97.5% |
|-----------|------------|------| | | | | | | |
| $\mu_0 dlPSI$ | -20.000 | 16.000 | | -17.813 | 1.414 | -20.612 | -17.811 | -15.076 |
| $\mu_1 dlPSI$ | 44.000 | 16.000 | | 43.475 | 1.706 | 40.134 | 43.490 | 46.785 |
| $\sigma^2$ | $-$ | $\infty$ | | 233.440 | 21.325 | 196.161 | 232.133 | 279.289 |
| $\beta_1$ | 1.000 | 5.000 | | 2.027 | 0.340 | 1.409 | 2.008 | 2.736 |
| $\beta_2$ | 0.000 | 5.000 | | -0.014 | 0.015 | -0.045 | -0.014 | 0.012 |
| $\beta_3$ | -1.000 | 5.000 | | -2.339 | 0.428 | -3.230 | -2.317 | -1.578 |
| $\beta_4$ | 0.000 | 5.000 | | 0.039 | 0.021 | 0.002 | 0.038 | 0.083 |

Table 2 - Portuguese Stock Index cycle chronologies

<table>
<thead>
<tr>
<th>PSI cycle reference dates</th>
<th>Duration (in months)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak</td>
<td>Trough</td>
</tr>
<tr>
<td>Peak-Trough</td>
<td>Trough-Peak</td>
</tr>
<tr>
<td>December 1988$^+$</td>
<td>July 1989</td>
</tr>
<tr>
<td>August 1990</td>
<td>May 1993</td>
</tr>
<tr>
<td>December 1994</td>
<td>January 1996</td>
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<tr>
<td>February 1999</td>
<td>November 1999</td>
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<tr>
<td>November 2000</td>
<td>August 2003</td>
</tr>
<tr>
<td>December 2007</td>
<td>September 2009</td>
</tr>
<tr>
<td>May 2010$^+$</td>
<td>January 2011$^+$</td>
</tr>
<tr>
<td>Average (6 cycles)</td>
<td>22</td>
</tr>
</tbody>
</table>

Notes: $^+$ indicates that the duration can be higher because the date of the respective peak or trough has been censored since they are out of the sample (January 1989-January 2011) and they are not known. December 1988 and January 2011 are assumed to be the reference (censored) dates, but the real peak/trough might be further away in the past or in the future, respectively. The censored durations are not considered in the computation of the averages.
Figure 1 - Evolution of the Portuguese Stock Index (January 1988 - January 2011)

Portuguese share prices index (base 2005=100)


Growth rate of the Portuguese share prices index


Source: OECD, Main Economic Indicators, February 2011.

Figure 2 - Transition Probabilities

Pr(S_t = 1 | S_{t-1} = 0, D_{t-1} = d)

Pr(S_t = 0 | S_{t-1} = 1, D_{t-1} = d)

Figure 3 - Probability of a Bull Market
Appendix

Figure A.1 - Means Estimates of $\mu_0$ and $\mu_1$

Figure A.2 - Kernel Density Estimates of $\mu_0$ and $\mu_1$

Figure A.3 - Correlograms of $\mu_0$ and $\mu_1$
Figure A.4 - Means Estimates of $\beta$

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