Insecticide control in a Dengue epidemics model

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Abstract. A model for the transmission of dengue disease is presented. It consists of eight mutually-exclusive compartments representing the human and vector dynamics. It also includes a control parameter (insecticide) in order to fight the mosquitoes. The main goal of this work is to investigate the best way to apply the control in order to effectively reduce the number of infected humans and mosquitoes. A case study, using data of the outbreak that occurred in 2009 in Cape Verde, is presented.

Keywords: optimal control, Dengue, control parameter
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INTRODUCTION

Dengue is a mosquito-borne infection mostly found in tropical and sub-tropical climates worldwide, mostly in urban and semi-urban areas. It can cause a severe flu-like illness, and sometimes a potentially lethal complication called dengue haemorrhagic fever. About 40% of the world's population are now at risk. The spread of dengue is attributed to expanding geographic distribution of the four dengue viruses and their mosquito vectors, the most important of which is the predominantly urban species *Aedes aegypti*. The life cycle of a mosquito presents four distinct stages: egg, larva, pupa and adult. In the case of *Aedes aegypti* the first three stages take place in or near water while air is the medium for the adult stage. The adult stage of the mosquito, in the urban environment, is considered to last an average of eleven days.

The paper is organized as follows. The next section presents a mathematical model of the interaction between human and mosquito populations. Then, the numerical experiments using different strategies for the insecticide administration are reported. In the last section the conclusions are presented.

THE MATHEMATICAL MODEL

Considering the work of [4], the relationship between humans and mosquitoes are now rather complex, taking into account the model presented in [2]. The novelty in this paper is the presence of the control parameter related to adult mosquito insecticide [6].

The notation used in our mathematical model includes four epidemiological states for humans (index h):

$S_h(t)$	susceptible (individuals who can contract the disease)
$E_h(t)$	exposed (individuals who have been infected by the parasite but are not yet able to transmit to others)
$I_h(t)$	infected (individuals capable of transmitting the disease to others)
$R_h(t)$	resistant (individuals who have acquired immunity)

It is assumed that the total human population (N_h) is constant, so, $N_h = S_h + E_h + I_h + R_h$. There are also other four state variables related to the female mosquitoes (index m) (the male mosquitoes are not considered in this study because they do not bite humans and consequently they do not influence the dynamics of the disease):

$A_m(t)$	aquatic phase (that includes the egg, larva and pupa stages)
$S_m(t)$	susceptible (mosquitoes that are able to contract the disease)
$E_m(t)$	exposed (mosquitoes that are infected but are not yet able to transmit to humans)
$I_m(t)$	infected (mosquitoes capable of transmitting the disease to humans)

In order to analyze the effects of campaigns to combat the mosquito, there is also a control variable:

$c(t)$ level of insecticide campaigns

Some assumptions are made in this model: the total human population (N_h) is constant, which means that we do not consider births and deaths; there is no immigration of infected individuals to the human population; the population is homogeneous; the coefficient of transmission of the disease is fixed and do not vary seasonally; both human and mosquitoes are assumed to be born susceptible; there is no natural protection; for the mosquito there is no resistant phase, due to its short lifetime.

The Dengue epidemic can be modelled by the following nonlinear time-varying state equations for Human Population and vector population, respectively:

$$\begin{cases} \frac{dS_h}{dt}(t) = \mu_h N_h - (B\beta_{mh}\frac{I_m}{N_h} + \mu_h)S_h \\ \frac{dE_h}{dt}(t) = B\beta_{mh}\frac{Im}{N_h}S_h - (\nu_h + \mu_h)E_h \\ \frac{dI_h}{dt}(t) = \nu_h E_h - (\eta_h + \mu_h)I_h \\ \frac{dR_h}{dt}(t) = \eta_h I_h - \mu_h R_h \end{cases} \quad \begin{cases} \frac{dA_m}{dt}(t) = \mu_b(1 - \frac{A_m}{K})(S_m + E_m + I_m) - (\eta_A + \mu_A)A_m \\ \frac{dS_m}{dt}(t) = -(B\beta_{hm}\frac{I_h}{N_h} + \mu_m)S_m + \eta_A A_m - cS_m \\ \frac{dE_m}{dt}(t) = B\beta_{hm}\frac{I_h}{N_h}S_m - (\mu_m + \eta_m)E_m - cE_m \\ \frac{dI_m}{dt}(t) = \eta_m E_m - \mu_m I_m - cI_m \end{cases} \tag{1}$$

with the initial conditions: $S_h(0) = S_{h0}$, $E_h(0) = E_{h0}$, $I_h(0) = I_{h0}$, $R_h(0) = R_{h0}$, $A_m(0) = A_{m0}$, $S_m(0) = S_{m0}$, $E_m(0) = E_{m0}$, $I_m(0) = I_{m0}$.

Notice that the equation related to the aquatic phase for the mosquito does not have the control variable c, because this kind of insecticide does not produce effects in this stage.

The parameters used in the model are:

N_h	total human population	μ_A	natural mortality of larvae (per day)
B	average daily biting (per day)	η_A	maturation rate from larvae to adult (per day)
β_{mh}	transmission probability from I_m (per bite)	$1/\eta_m$	extrinsic incubation period (in days)
β_{hm}	transmission probability from I_h (per bite)	$1/\nu_h$	intrinsic incubation period (in days)
$1/\mu_h$	average lifespan of humans (in days)	m	female mosquitoes per human
$1/\eta_h$	mean viremic period (in days)	k	number of larvae per human
$1/\mu_m$	average lifespan of adult mosquitoes (in days)	K	maximal capacity of larvae
μ_b	number of eggs at each deposit per capita (per day)		

The Figure 1 shows the relation between human and mosquito and the corresponding model parameters.

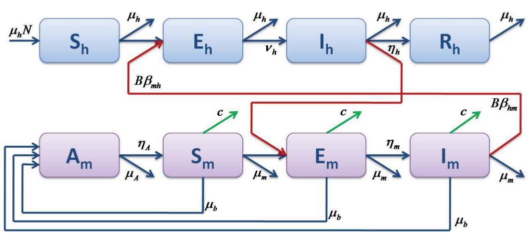

FIGURE 1. Interaction between human and mosquito

NUMERICAL EXPERIMENTS

The values related to humans originate from data collected in an infected period in Cape Verde [1]. However, as it was the first outbreak that happened in the archipelago, it was not possible to collect any data for the mosquito. Thus, for the *Aedes aegypti* we have selected information from Brazil where dengue is already a long known reality [8, 9].

The numerical tests were carried out using Scilab [7] with the following values [8, 9]: $N_h = 480000$, $B = 1$, $\beta_{mh} = 0.375$, $\beta_{hm} = 0.375$, $\mu_h = 1/(71 \times 365)$, $\eta_h = 1/3$, $\mu_m = 1/11$, $\mu_b = 6$, $\mu_A = 1/4$, $\eta_A = 0.08$, $\eta_m = 1/11$, $\nu_h = 1/4$, $m = 6$, $k = 3$, $K = k \times N_h$. The initial conditions for the problem were: $S_{h0} = N_h - E_{h0} - I_{h0}$, $E_{h0} = 216$, $I_{h0} = 434$, $R_{h0} = 0$, $A_{m0} = k \times N_h$, $S_{m0} = m \times N_h$, $E_{m0} = 0$, $I_{m0} = 0$. The final time was $t_f = 84$ days.

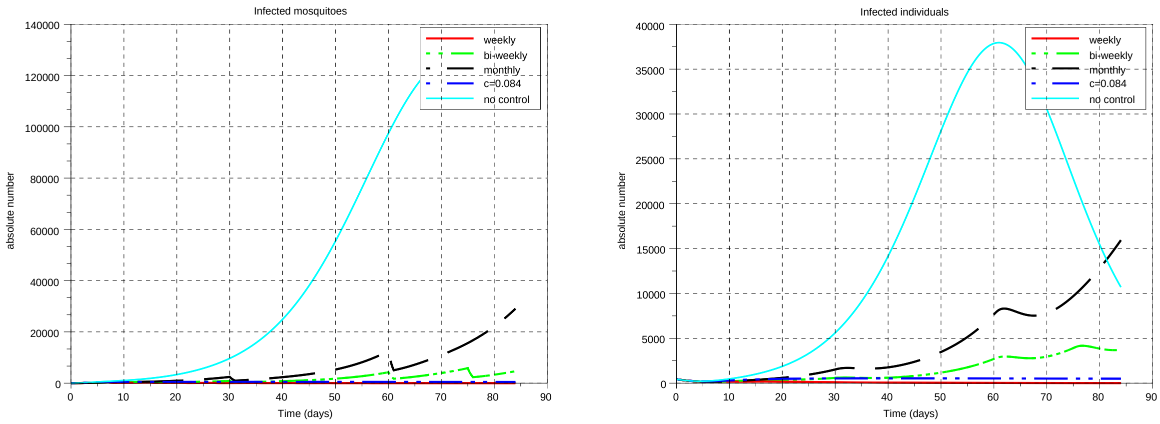

FIGURE 2. Infected mosquitoes and individuals

In the literature it has been proved that a disease-free equilibria (DFE) is locally asymptotically stable, whenever a certain epidemiological threshold, known as the *basic reproduction number*, is less than one. The *basic reproduction number* of the disease represents the expected number of secondary cases produced in a completed susceptible population, by a typical infected individual during its entire period of infectiousness [3]. In a recent work [5] it was proved that if a constant minimum level of insecticide is applied ($c = 0.084$), it is possible to maintain the basic reproduction number below unity, guaranteing the DFE.

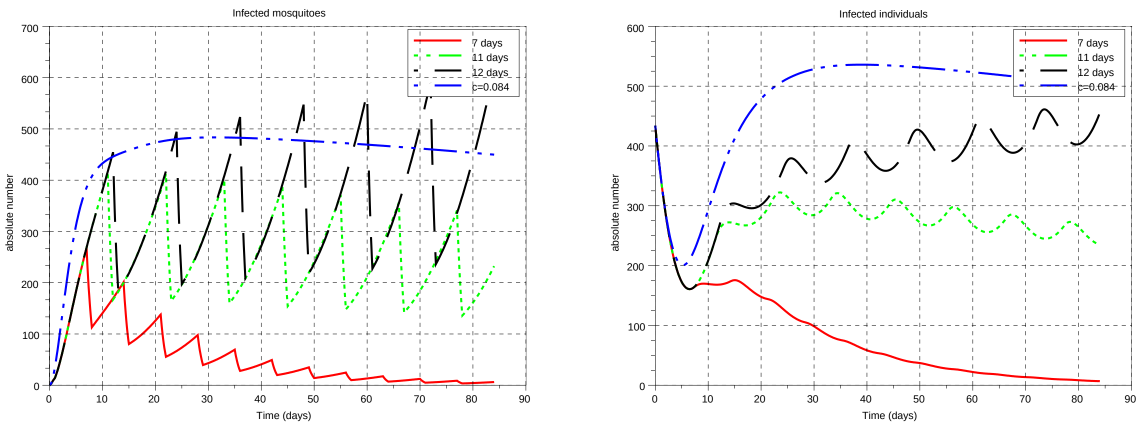

FIGURE 3. Infected mosquitoes and individuals *vs* periodicity

To solve the system (1), in a first step, several strategies of control application were used: three different frequencies (weekly, bi-weekly and monthly) for control application, constant control ($c = 0.084$ in [5]) and no control ($c = 0$). The weekly/bi-weekly/monthly frequency means that during one day per week/bi-week/month the whole (100%)

capacity of insecticide ($c = 1$) is used during all day. Constant control strategy ($c = 0.084$) consists in the application of 8.4% capacity of insecticide 24 hours per day all the time (84 days). In this work, the amount of insecticide is an adimensional value and must be considered in relative terms.

Figure 2 shows the results of these strategies regarding infected mosquitoes and individuals. Without control, the number of infected mosquitoes and individuals increases expressively.

Realizing the influence of the insecticide control, further tests were carried out to find the optimum periodicity of administration which, from gathered results, must rest between one and two weeks. The second phase of numerical tests, Figure 3, considers four situations: 7 days, 11 days, 12 days and continuously $c = 0.084$. To guarantee the DFE, the curves must remain below the one corresponding to $c = 0.084$.

The amount of insecticide, and when to apply it, are important factors for outbreak control. Table 1 reports the total amount of insecticide used in each version during the 84 days.

TABLE 1. Insecticide cost

	7 days	11 days	12 days	15 days	30 days	$c = 0.084$
insecticide amount	12	8	7	6	3	7.056

CONCLUSIONS

The numerical tests conclude that the best strategy for the infected reduction is the weekly administration, however it is also the most expensive one (insecticide cost). The best result obtained is between 11 and 12 days, with the insecticide amount in the closed interval from 7 to 8, confirming the amount for $c = 0.084$ in [5]. The 11 or 12 days between applications can be directly related to the span of adult stage for the mosquitoes, an average of eleven days in an urban environment.

In this work the insecticide administration time was considered continuous in each day (24 hours per day). As a future work, an optimization problem will be formulated and solved in order to find the best plan in terms of periodicity and duration (limited number of hours per day). The plan must consider the advantages to apply the insecticide only during the night.

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