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ROLE OF FRACTIONAL CAKE COMPOSITION IN CAKE RESISTANCE

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Abstract – The binary mixture model was applied in order to provide an explanation of the discrepancy between the calculated and experimentally measured specific cake resistance in TiO_2 cakes. The case under consideration shows clearly that, even where physicochemical factors influence the particle size distribution, for some bimodal dispersed systems where the ratio of larger diameter particles to those of smaller diameter is greater than 4, the specific cake resistance can be defined by using the particle mixture approach. Consideration of tortuosity as a variable parameter dependent on porosity may improve the degree of fit between the cake resistance modelling and experimentally measured results.
Keywords: Microfiltration; porosity; tortuosity; specific cake resistance.

INTRODUCTION

The appropriate description of flow phenomena in porous media (membrane, filter and filter cake, granular beds, etc.) is important for the design and control of separation or mass transfer processes in industry.

Modelling of these phenomena range from a pure hydraulic approach to entirely physico-chemical models[1-7]. The role of hydraulic and physicochemical factors in a dispersed separation system depends on a characteristic system scale. In colloidal systems, for example, a decreasing scale of particles increases the role of physicochemical factors. However, when the dispersed system includes particles of significant size diversity, both approaches must be carefully analysed.

If one considers a dispersed system then, usually, physicochemical factors are responsible for aggregation of primary particles or for the destruction of aggregates of primary particles. They also define the durability of aggregates in the dispersion as well as in the filter cake, which in turn affects the cake compressibility.

In any case, the cake hydraulic resistance (specific cake resistance, α) depends on the particle fraction composition in the deposit[8-16]. When physicochemical factors are dominant, the cake particle composition seems to be of secondary importance. However, where the particle size distribution is broad the cake particle composition might become a significant parameter in determining the specific cake resistance properties.

RESULTS AND DISCUSSION

Bowen and Goenaga[7] studied the crossflow microfiltration of TiO_2 particles through an inorganic membrane (pore size 0.2 μm), with a filtration pressure of 34.5 kPa. The authors showed that the permeation rate depends strongly on the pH and on the ionic strength of the supporting electrolyte. As a final conclusion, it was pointed out that this variation cannot be explained simply in terms of particle size distribution and it was suggested that an electro-kinetically enhanced back-diffusion mechanism might operate.

Due to the bi-disperse particle size distribution used in the experiment, as is shown in Figure 1 and because of the wide gap between each average particle size fraction (around 5/0.5 μm), it is possible to show that specific cake resistance can be satisfactorily described in terms of a binary particle mixture[16].

For the calculation of specific cake resistance, the following assumptions were used:

1. The dispersed phase was considered as a binary mixture. The particle size in each fraction was defined as the size corresponding to the maxima of the particle size distribution curve of a bimodal distribution. The fraction particle

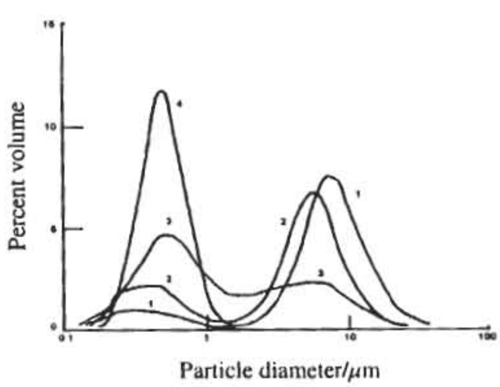

Figure 1: Variation in the size distribution of titanium dioxide dispersions with ionic strength at pH 8. (1) 10^{-1}M, (2) 10^{-2}M, (3) 10^{-3}M, (4) 10^{-4}M (Adapted from [7]).

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sizes were determined from a previous study[7].

2. The volume fraction of large particles in the binary mixture, x_D, was calculated using the proportion between large and small particles corresponding to the maximum points of the bimodal distribution.
3. The only dispersions that were considered were those represented on the figures. Table 1 shows dispersion data as measured by Bowen and Goenaga[7] together with calculations based on the binary mixture approach. It should be noted that the measured membrane resistance, R_m, assumes the absence of progressive membrane clogging.

Image modelling and experiments using binary mixtures of spherical particles of different sizes show that the tortuosity of pore channels is not constant and depends on the packing porosity[14,17,18]. Therefore, for comparison, the specific cake resistance was calculated using two types of equations:

$$\alpha_0 = 180\frac{1-\varepsilon}{\rho_s d_{av}^2 \varepsilon^3} \qquad (1)$$

and

$$\alpha_1 = \frac{36K(1-\varepsilon)}{\rho_s d_p^2 \varepsilon^3} = \frac{36K_0 T^2 (1-\varepsilon)}{\rho_s d_p^2 \varepsilon^3} \qquad (2)$$

where, ε is the cake porosity; ρ_s is the solid phase density, d_{av} is the average mixture particle size, T is the tortuosity, $K = K_0 T^2$ is the Kozeny's coefficient and K_0 is a shape coefficient.

Equation (1) was used by Bowen and Goenaga[7] to calculate the porosity of a primary particle deposit, i.e., $\varepsilon = 0.35$. It must be noted that Bowen and Goenaga[7] obtained this result using mono-disperse particles in a 10^{-4} solution. The equation these authors used has a numerical Kozeny's coefficient of $180 = 36K = 36K_0T^2$ and normally fits well with mono-sized granular beds (for granular beds $K = 4.2 \div 5.0$)[19], where K_0 may be assumed to equal 2.0 (Happel and Brenner)[20].

Equation (2) differs from Equation (1) by assuming that the tortuosity is variable, which, for granular beds, may be represented by a power function[17,18,21,22].

$$T = 1/\varepsilon^n \qquad (3)$$

where n = 0.4 – 0.5 and the power coefficient is 0.4 for spherical particles. Since TiO_2 particles are not spherical, tortuosity must be more affected by the porosity and it has been assumed in this work that n = 0.5, which means that $T = \varepsilon^{0.5}$

The average particle size of the binary mixture was calculated by the formula:

$$\frac{1}{d_{av}} = \frac{x_D}{D} + \frac{1-x_D}{d} \qquad (4)$$

where D is the large particle size and d the small particle size. All values used in Equation (4) are presented in Table 1 together with those calculated by Equations (1) and (2).

The scattering between the measured specific cake resistance, α, and the calculated values, α_0 and α_1, is shown in Figure 2.

As can be seen from Table 1, the specific cake resistance, calculated using the assumption of a binary cake composition is in reasonable agreement with that determined experimentally. Nevertheless, the best fit is undoubtedly obtained for Equation (2), when the tortuosity is considered as a function of the mixture porosity and the average diameter is calculated by Equation (4). By calculating the sum of squares of the different calculation approaches:

$$SS_0 = \sum (\alpha - \alpha_0)^2 \qquad (5)$$

and

$$SS_1 = \sum (\alpha - \alpha_1)^2 \qquad (6)$$

we have SS_0 = 22.9877 x 10^{22} and SS_1= 2.28066 x 10^{22} (m kg^{-1})^2, and thus SS_0 / SS_1 = 10.05.

These results imply that Equation (2) provides a better fit with the experimental values than Equation (1). It may be surprising that, by considering the porosity value 0.35 throughout the range of particle sizes, Equation (2) fits so well. This may probably mean that porosity remains approximately constant for all ranges of particle sizes, even in the case of aggregates. The latter case might be more affected by tortuosity, which may explain the better fit of Equation (2).

Data as measured by Bowen and Goenaga[7]					Analysis based on binary mixture approach					
pH	I(M)*	R_m (m^{-1})	u^*,(μm s^{-1})/(V cm^{-1})	α (m kg^{-1})	α_1 (m kg^{-1})	α_0 (m kg^{-1})	x_D	d (μm)	D (μm)	D_{av} (μm)
8.0	10^{-4}	2.85x10^{11}	- 3.38	29.9x10^{11}	29.26x10^{11}	25.6x10^{11}	0.0	0.50	-	0.50
8.0	10^{-3}	4.02x10^{11}	- 3.21	13.6x10^{11}	14.4x10^{11}	12.6x10^{11}	0.333	0.50	5.00	0.71
8.0	10^{-2}	2.57x10^{11}	- 2.87	4.8x10^{11}	5.26x10^{11}	4.6x10^{11}	0.740	0.36	5.90	1.18
8.0	10^{-1}	2.26x10^{11}	- 2.73	3.5x10^{11}	2.47x10^{11}	2.16x10^{11}	0.865	0.30	6.60	1.72
4.0	10^{-2}	2.20x10^{11}	- 0.12	3.63x10^{11}	3.17x10^{11}	2.77x10^{11}	0.848	0.30	5.55	1.52
6.0	10^{-2}	2.72x10^{11}	- 1.29	3.65x10^{11}	3.31x10^{11}	2.9x10^{11}	0.810	0.36	5.55	1.48
8.0	10^{-2}	2.57x10^{11}	- 2.88	4.8x10^{11}	4.8x10^{11}	4.2x10^{11}	0.758	0.36	5.55	1.24

*I(M) = ionic strength of KNO_3, M.

Table 1: Comparison between the cross flow filtration data and the variation in particle mobility calculated by Bowen and Goenaga[7] and the values calculated in this study using the two models.

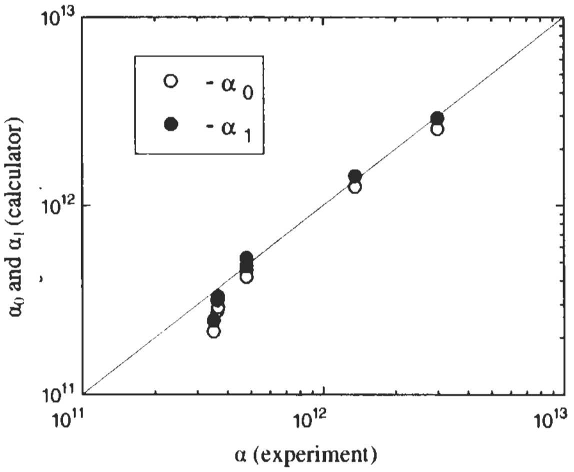

Figure 2: Comparison of measured values of α with calculated α_0 and α_1 (m kg^{-1}).

CONCLUSIONS

The additional parameter, *n*, which was introduced in the definition of tortuosity as a function of porosity, does not justify *per se* the improvement in fit. As a matter of fact, as tortuosity becomes a non-linear function of porosity, the term *T* disappears from the equations and the total number of parameters and variables remains the same.

Mota *et al* [16] showed that the permeability and the corresponding cake resistance in spherical binary mixtures are affected by increases in the mean size ratio of the particles. The present study has analysed the experimental filtration of real bimodal TiO_2 particles in the light of previous studies that investigated regular spherical mixtures

The case considered here indicates that, even when physicochemical factors influence the particle size distribution, for some dispersed systems that have a wide gap in their average fractional particle sizes ($D/d > 4$), the specific cake resistance can be modelled with the help of a particle mixture approach. It may also be concluded that the consideration of tortuosity as a variable parameter dependent on porosity may improve the modelling of cake resistance and its conformity to the values measured by experimentation.

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NOMENCLATURE

d	Small size particle diameter (m)
d_{av}	Average particle size in binary mixture (m)
D	Large particle size (m)
K	Kozeny's coefficient
K_0	Shape coefficient depending on a cross-section capillary pore shape
R_m	Membrane hydraulic resistance (m^{-1})
T	Tortuosity
x_D	Volume fraction of large particle in binary mixture (in cake)
α	Average specific cake mass resistance (m kg^{-1})
ε	Porosity
ρ_s	Solid phase density (kg m^{-3})
u	Electrophoretic mobility ((μm s^{-1})/(V cm^{-1}))

REFERENCES

1. Rushton A., Ward A.S. and Holdich R.G., 1996, *Solid-Liquid Filtration and Separation Technology*, VCH, Germany.
2. Brik M.T. and Tsapuk E.A., 1989, *Ultrafiltration*, Naukova Dumka, Kiev, pp. 190-191.
3. Havsteen B.H., 1993, Anomalous filtration, *Advances in Colloid and Interface Science*, **45**, 79-213.
4. Ho W.S.W. and Sirkar K.K., 1992, *Membrane Handbook*, WNR, New York.
5. Rautenbach R. and Albrecht R., 1989, *Membrane Processes*, John Wiley Limited, U.K.
6. Probstein R.F., 1989, *Physicochemical Hydrodynamics: An Introduction*, Butterworth Publishers, Stoneham, U. S.A.
7. Bowen W.R. and Goenaga X., 1990, Properties of microfiltration membranes. Part 3. Effect of physicochemical conditions on crossflow microfiltration at aluminium oxide membranes, *Advances in Separation Processes, IChemE Symposium Series*, **118**, 107-118.
8. Abe E. and Hirosue H., 1982, Porosity estimation of a mixed cake in body filtration, *Journal of Chemical Engineering of Japan*, **15**(6), 490-493.
9. Yu A.B. and Standish N., 1991, Estimation of the porosity of particle mixtures by a linear-mixture packing model, *Industrial Engineering Chemistry Research*, **30** (6), 1372-1385.
10. Yu A.B., Standish N. and McLean A., 1993, Porosity calculation of binary mixtures of nonspherical particles, *Journal of American Ceramics Society*, **76**(11), 2813-2816.
11. Li Y. and Park C.-W., 1998, Permeability of packed beds filled with polydisperse spherical particles, *Industrial Engineering Chemistry Research*, **37**(5), 2005-2011.
12. Meeten G.H. and Sherwood J.D., 1994, The hydraulic permeability of bentonite suspensions with granular inclusions, *Chemical Engineering Science*, **49**(19), 3249-3256.
13. Okoh B.O., 1989, Porosity and permeability as a function of fraction of filter aid, *Fluid/Particle Separation Journal*, **2**(1), 37-43.
14. Mota M., Teixeira J.A., Bowen W.R. and Yelshin A., 2000, Effect of tortuosity on transport properties of mixed granular beds, *Proceedings of 8th World Filtration Congress*, 3rd-7th April 2000, Vol. 1, The Filtration Society, Brighton, UK, pp. 57-60.
15. Yelshin A., Mota M. and Teixeira J., 1997, Porous media behaviours modelling and analysis in separation processes, *Proceedings of Filtech Europa 97 Interna-*

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tional Conference, October 14th – 16th 1997, The Filtration Society, Düsseldorf, Germany, pp. 327-334.

16. Mota M., Teixeira J.A. and Yelshin A., 2001, Binary spherical particle mixed beds porosity and permeability relationship measurement, *Transactions of the Filtration Society*, **1**(4), 101-106.
17. Mota M., Teixeira J.A. and Yelshin A., 1999, Image analysis of packed beds of spherical particles of different sizes, *Separation and Purification Technology*, **15**, 59-68.
18. Mota M., Teixeira J.A. and Yelshin A., 1998, Toruosity in bioseparations and its application to food processes, *Proceedings of the 2nd European Symposium on Biochemical Engineering Science*, Porto, 16th-19th September, *1998*, Feyo de Azevedo, Ferreira, E., Luben, K. and Osseweijer P. (Eds.), Porto, Portugal, pp. 93-98.
19. Chiang S.-H. and He D., 1993, Filtration and dewatering: theory and practice, *Filtration and Separation*, **6**(2), 64-83.
20. Happel J. and Brenner H., 1965, *Low Reynolds Number Hydrodynamics*, Prentice-Hall.
21. Currie J.A., 1960, Gaseous diffusion in porous media. Part 2. - Dry granular materials, *British Journal of Applied Physics*, **11**, 318-324.
22. Satterfield C. N., 1980, *Heterogeneous Catalysis in Practice*, McGraw-Hill.

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